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**multi-Risk sciEnce for resilienT commUnities undeR a changiNclimate**

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## 1. Technical references

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\* PU = Public

PP = Restricted to other programme participants (including the Commission Services)

RE = Restricted to a group specified by the consortium (including the Commission Services)

CO = Confidential, only for members of the consortium (including the Commission Services)

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## Document history

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0.2	18.03.2026	Alberto Montanari (UNIBO), Salvatore Grimaldi (Università della Tuscia), Elena Volpi (Università RomaTre), Marco Marani (UNIPD), Marco Borga (UNIPD)	Critical review and proofreading

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## 2. Abstract

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Based on the activities carried out in Task 8.3.2, this deliverable introduces a methodology for the generation of climate and weather scenario at the local level. The distinguishing feature of the proposed approach is the integration of physically based climate models with an advanced stochastic weather generator, allowing to preserve the statistical features of the historical climate and incorporate the climate change projections delivered by climate models.

The present deliverables includes two papers submitted for publication, describing the procedure and its application to the Arno River Basin.

The methodology is an innovative product of the RETURN project, allowing a seamless and synergetic integration of models for producing an innovative information to support technical design of risk mitigation policies and projects at the local level.

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## Table of contents

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1. Technical references .....	2
Document history .....	3
2. Abstract.....	4
Table of contents .....	5
3. Introduction .....	6
4. Bridging Flexibility and Usability: Configuring CoSMoS-2s for Operational Rainfall Simulation in the Arno River.....	7
5. Multisite Rainfall Simulation in the Arno River Basin in Italy.....	23

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### 3. Introduction

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The RETURN methodology for generating local climate scenarios relies on the integration of the CoSMoS model (version CoSMoS-2S) and the projections for statistics of future climate delivered by any selected climate model.

The deliverable describes the application of the CoSMoS model, which leads to estimation of its parameters to match the statistical behaviours of the historical climate. The integration of the projections of climate models can be attained by accordingly varying the CoSMoS parameters.

The CoSMoS model was introduced by Papalexiou (2018). It is capable of simulating intermittent hydroclimatic processes such as precipitation. The marginal probability distribution of the data can be continuous, mixed-type, discrete or binary. These behaviours need to be reproduced as well as joint probability distributions and autocorrelation function. By preserving these features we ensure the generation of synthetic time series with high degree of realism and accuracy, therefore giving the opportunity to identify extreme events beyond what has been observed. Furthermore, by modifying the parameters of the model according to predicted climate change, one can simulate the behaviours of expected future climate therefore mitigating surprise.

The basic assumption underlying CoSMoS is that any process -stationary or cyclostationary- can be generated by transforming a specific "parent" Gaussian process. In the case of a multivariate process, each process within the group has a corresponding parent Gaussian process and analytical transformations of both auto and cross-correlation functions are employed. By exploiting the relationship with the parent Gaussian processes CoSMoS is able to simulate a wide range of hydroclimatic phenomena, spanning from intermittent processes like precipitation, heavy-tailed processes such as river discharge, processes with bounded marginals like relative humidity as well as discrete (rain days/month) and binary cases (wet/dry conditions).

The most recent extension of CoSMoS by Papalexiou (2022), preserves the main statistical characteristics of the hydrological processes such as intermittency, marginal distributions and correlation functions. Within this approach, it is suggested that any observed intermittent time series with a specific marginal distribution, is transformed in a time series with a mixed-uniform marginal distribution. In particular, this approach enables that a given time series with a specific probability of zeros and a specified marginal distribution is transformed in a time series with a one-parameter mixed-uniform distribution.

The next section of the deliverable presents a copy of a paper submitted to a scientific journal describing CoSMoS-2S and its application to the Arno River. The final section presents a second paper (in Italian) providing more details on the rainfall simulation for the Arno River Basin.

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## **4. Bridging Flexibility and Usability: Configuring CoSMoS-2s for Operational Rainfall Simulation in the Arno River**

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1 **Bridging Flexibility and Usability: Configuring CoSMoS-2s for Operational Rainfall Simulation in the Arno River**  
2 **Basin**

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10  
11 **Abstract**

12 Rainfall simulation models are essential for hydrological studies, supporting applications in flood control,  
13 urban planning, agriculture, and climate change adaptation. A key challenge is developing models that  
14 effectively capture rainfall intermittency while remaining computationally efficient and practical for  
15 operational use. One recently introduced approach, the two-state intermittent model CoSMoS-2s, simulates  
16 rainfall time series while preserving arbitrary marginal distributions and autocorrelation structures. However,  
17 its practical implementation requires robust and consistent parameterization to ensure reliability. Here, we  
18 present a systematic configuration of CoSMoS-2s that minimizes parameter requirements while maintaining  
19 strong performance, improving its ease of calibration and operational applicability. To validate its real-world  
20 utility, we test the model using a large database of daily rainfall time series from seventy rain gauges with  
21 contemporaneous observations spanning 20 years—a scenario where data limitations pose a significant  
22 challenge. The results demonstrate that CoSMoS-2s effectively reproduces key rainfall characteristics,  
23 including wet and dry spell durations, dry-day probabilities, statistical properties of nonzero rainfall amounts,  
24 and extreme rainfall behavior. Compared to previous implementations, this minimal parameterization  
25 enhances usability without compromising accuracy. These findings indicate that CoSMoS-2s is a robust and  
26 versatile tool for hydrological modeling, particularly in data-scarce regions where reliable rainfall simulations  
27 are critical.

28 **Keywords:** Stochastic simulation model, Rainfall Simulation, Daily rainfall time series, Complete Stochastic  
29 Modelling Solution (CoSMoS)

30 **1. Introduction**

31 Rainfall simulation models are essential in climatology, hydrology, meteorology, and environmental sciences,  
32 supporting risk assessment, infrastructure planning, water resource management, and climate change impact  
33 studies (e.g., Beck et al., 2020; Clark et al., 2004; Iizumi et al., 2017; Kim et al., 2021; Li et al., 2021;  
34 Papalexiou, 2018, 2022). Stochastic models can help design resilient infrastructure, optimize water allocation,  
35 and assess flood and drought risks by generating realistic rainfall scenarios. These models are used in  
36 hydrological simulations, providing precipitation inputs for rainfall-runoff analysis, reservoir operations,  
37 stormwater management, etc. For example, a key application is stress-testing infrastructure under different  
38 precipitation scenarios to ensure long-term functionality. Stochastic models can also be linked with covariates,  
39 such as temperature, and be used in climate change studies to assess shifts in intensity, frequency, and duration  
40 in future rainfall patterns; this in turn, can inform adaptation strategies in water and food security, flood  
41 mitigation, etc. (e.g., Chandler et al., 2007; Singer et al., 2018; Rios Gaona et al., 2024, Fisher et al. 2025).  
42 Their ability to generate multiple synthetic rainfall sequences allows also ensemble forecasting which is pivotal  
43 in robust uncertainty analysis in hydrology.

44 In general, rainfall models aim to capture the inherent variability of precipitation and different frameworks  
45 have been used (Volpi et al., 2024). Cluster-based models, such as the Neyman-Scott (Neyman & Scott, 1958)  
46 and Bartlett-Lewis (Rodríguez-Iturbe et al., 1987) rectangular pulse models, represent storms as clusters of  
47 pulses with random intensities and durations. These models use Poisson-distributed storm arrivals, with  
48 individual cells following specific distributions. Various extensions have improved their flexibility by refining  
49 storm durations, cell intensities, and temporal structures (Cowpertwait, 1991; Wheeler et al., 2005). Another  
50 widely used approach is Markov Chain modeling (Gabriel & Neumann, 1957, 1962), which simulates rainfall  
51 occurrence based on wet/dry state transitions. Higher-order and multi-state variations (e.g., Stern & Coe, 1984)

enhance the representation of wet/dry sequences, while hybrid models integrate continuous distributions to improve rainfall intensity simulations (e.g., Wilks & Wilby, 1999).

Multifractal approaches also exist targeting to capture scale-invariant properties of precipitation. Multiplicative cascade models (Schertzer & Lovejoy, 1987) have been applied to generate fine-resolution rainfall fields while preserving variability across scales. These methods have been used for stochastic disaggregation of daily rainfall (Gaume et al., 2007; Molnar & Burlando, 2005; Cappelli et al. 2024b) and for space-time modeling of precipitation extremes (Veneziano et al., 2006). Disaggregation techniques further refine rainfall simulations by ensuring consistency between different temporal scales. Recent schemes, such as DiPMaC (Papalexiou et al., 2018), explicitly preserve probability distributions and correlations while accommodating non-stationary characteristics, enhancing the accuracy of fine-scale precipitation modeling. Beyond these approaches, Gaussian-based models have been extensively applied in rainfall simulation. Autoregressive moving average processes (Box & Jenkins, 1970; Matalas, 1967) have been used to generate synthetic time series, while threshold-based transformations enable their use for intermittent rainfall (Bárdossy & Plate, 1992; Glasbey & Nevison, 1997). Copula-based methods further enhance dependence structures across sites (Bárdossy & Pegram, 2009; Serinaldi, 2009), and correlation transformation techniques allow for improved representation of rainfall intermittency and marginal distributions (Papalexiou, 2022).

Here we focus on the CoSMoS (Complete Stochastic Modelling Solution), introduced by Papalexiou (2018), which provides a unified framework for the stochastic modeling of hydroclimatic processes, with several extensions developed in recent years (Papalexiou, 2022; Papalexiou, et al., 2023; Cappelli et al., 2024a). Specifically, we use the two-state intermittent model (CoSMoS-2s) (Papalexiou, 2022), which integrates a binary process for rainfall occurrence with a continuous process for rainfall amounts. This approach enables the reproduction of both linear and nonlinear autocorrelation structures (ACS), as well as the intermittency and marginal distribution of nonzero values, capturing rainfall characteristics across multiple scales. CoSMoS-2s seamlessly integrates occurrence or intensity, ensuring that wet/dry sequences and nonzero rainfall amounts are realistically represented while allowing for total flexibility in the choice of arbitrary marginal distributions and correlation structures. This dual control over intermittency and intensity enhances adaptability across different climatic and hydrological settings but also requires careful parameterization.

The strength of CoSMoS-2s lies in its ability to balance flexibility with practical implementation, yet this also introduces challenges. While its adaptability makes it suitable for diverse climatic and hydrological settings, it also shifts the burden of selecting appropriate modeling choices to practitioners. A highly flexible model allows for extensive customization, but without proper guidance, this flexibility can become a limitation rather than an advantage. Theoretical advancements provide a broad range of modeling choices yet applying them effectively requires rigorous evaluation and careful parameterization to ensure consistency with real-world rainfall characteristics. This study addresses that gap by demonstrating how CoSMoS-2s can be applied in a parsimonious yet robust manner, making it both practical and reliable. To assess its real-world applicability, we conduct a large-scale case study using 70 raingauges within the Arno River Basin, where the relatively short 20-year time series present a challenging test for model validation. By evaluating its ability to reproduce key rainfall properties, we aim to bridge the gap between theoretical innovation and practical hydrological modeling, highlighting CoSMoS-2s as a powerful yet accessible tool for rainfall simulation.

The paper is organized as follows: Section 2 presents the theoretical background of the rainfall simulation model and details the implementation strategies. Section 3 provides a brief overview of the case study area. Section 4 thoroughly evaluates the model’s performance, and Section 5 summarizes the key findings and presents the conclusions.

## 2. Summary of the CoSMoS-2s Model

In this section, we briefly summarize the specifics of the CoSMoS-2S model, focusing on the modelling choices and its calibration and implementation. Section 2.1 outlines the theoretical foundations of the framework, while Section 2.2 provides a step-by-step guide to its practical application. Finally, Section 2.3 discusses implementation strategies and calibration techniques to ensure accurate representation of real-world hydroclimatic processes.

### 2.1. Conceptual Note on the Modelling Framework

In general, the CoSMoS-2s model provides a framework for simulating intermittent hydroclimatic processes, such as rainfall at any temporal scale. The model reproduces intermittent rainfall through a combination of two distinct stochastic components:

1. Rainfall occurrence is modeled as a binary process, generating a sequence of ones and zeros to distinguish between dry (0, no rainfall) and wet (1, nonzero rainfall) conditions. This process has its own dependence

107 structure (meaning past wet/dry states influence future states), designed to capture the persistence and  
 108 clustering of dry and wet spells over time while accurately reproducing their frequencies.  
 109 2. The magnitude of rainfall is modeled as a separate stochastic process with a flexible continuous  
 110 probability distribution and an associated dependence structure, ensuring realistic temporal variability and  
 111 statistical consistency with observed precipitation patterns.

112 The CoSMoS-2s model is an integral component of the broader CoSMoS framework, designed to simulate  
 113 hydroclimatic processes across various scales and dimensions, including univariate, multivariate, and multisite  
 114 settings, as well as spatiotemporal random fields. These fields can range from purely static to highly dynamic  
 115 systems exhibiting complex anisotropies and advection components. As previously mentioned, CoSMoS-2s is  
 116 specifically tailored for intermittent processes such as precipitation, offering multiple modeling configurations  
 117 that result in a diverse set of variants. Both the binary and continuous components can be driven by different  
 118 parent processes, including Gaussian, t-copula, or other copula-based structures. This flexibility allows, for  
 119 example, the generation of intermittent time series where nonzero values preserve the dependence structure of  
 120 any chosen copula.

121 Notably, CoSMoS-2s enables the analytical approximation of binary sequences with arbitrary correlation  
 122 structures and offers a rank-based formulation that preserves Spearman's rank correlations. This rank-based  
 123 approach can be applied analytically, eliminating the need for numerically solving integral transformations or  
 124 fitting correlation functions. In this study, we explore and apply this variant to demonstrate its operational  
 125 effectiveness.

## 126 2.2. Step-by-Step Implementation of the Rank-Based CoSMoS-2s

127 The first state of the CoSMoS-2s model involves creating a binary time series that represents wet (rain) and  
 128 dry (no rain) periods while preserving the temporal dependencies observed in real rainfall data. The process  
 129 consists of the following steps:

- 130 1. Extract the binary sequence from observed data: Convert the observed precipitation time series into a  
 131 binary sequence by replacing all nonzero values with 1 and all zero values with 0.
- 132 2. Assess key characteristics of the binary process: Determine the probability of zero  $p_0$  of the binary time  
 133 series and estimate the empirical ACS  $\hat{\rho}_B(\tau)$  which characterizes how dry and wet states persist over time.  
 134 Note that for a binary time series the Pearson autocorrelation and Spearman's rank correlation are the  
 135 same.
- 136 3. Optionally, use the  $\hat{\rho}_B(\tau)$  to fit a parametric ACS to describe parsimoniously the temporal dependence of  
 137 wet/dry states.
- 138 4. Compute parameters for the parent Gaussian process: For the estimated  $p_0$  find  $c_{p_0}$  using

$$139 \quad c_{p_0} = a \left( \sqrt{p_0(1-p_0)} + (p_0(1-p_0))^b \right) \quad (1)$$

with  $a = 0.81$  and  $b = 0.10$ . Then compute the ACS of the parent Gaussian process from

$$140 \quad \rho_Z(\tau) = 1 - (1 - \rho_B(\tau)^{c_{p_0}})^2 \quad (2)$$

where  $\rho_B(\tau)$  are the estimated autocorrelations (for any lag) of the binary time series.

- 141 5. Generate a parent Gaussian time series: Simulate a standard Gaussian time series  $z(t)$  using an  
 142 autoregressive (AR) model of sufficiently large order to reproduce  $\rho_Z(\tau)$ .
- 143 6. Transform to a binary time series: Convert the Gaussian generated values  $z(t)$  to a binary sequence  $b(t)$   
 144 by applying the threshold transformation:  $b(t) = 0$  if  $z(t) \leq z_{p_0}$ , and  $b(t) = 1$  if  $z(t) > z_{p_0}$  where  $z_{p_0} =$   
 145  $-\sqrt{2}\text{erfc}^{-1}(2p_0)$ .

146 This process generates a binary time series that matches the observed probability of zero ensuring the same  
 147 frequency of wet and dry periods. Additionally, it preserves the temporal dependence structure, closely  
 148 replicating the clustering of wet and dry states over time.

149 The second step in the rank-based CoSMoS-2s model involves generating a continuous time series  
 150 representing rainfall amounts on wet days. This process ensures that the generated values follow the observed  
 151 rainfall distribution while reproducing the Spearman's rank autocorrelation  $\rho_S(\tau)$ . The steps are as follows:

- 152 1. Estimate the autocorrelations of the continuous process: Compute the empirical Spearman's  
 153 autocorrelation  $\hat{\rho}_S(\tau)$  using positive rainfall values from observed data.

- 154 2. Optionally, use the  $\hat{\rho}_S(\tau)$  to fit a parametric ACS  $\rho(\tau)$  to describe the temporal dependence of rainfall  
 155 magnitudes instead of using the empirical values.  
 156 3. The parent Gaussian ACS  $\rho_Z(\tau)$  is obtained using

$$\rho_Z(\tau) = 2 \sin\left(\frac{\pi}{6}\rho_S(\tau)\right) \quad (3)$$

157 where  $\rho_S(\tau)$  are the estimated autocorrelations (for any lag) of the binary time series.

- 158 4. Fit a probability distribution to nonzero rainfall values: Estimate a parametric cumulative distribution  
 159 function (CDF)  $F_X(x)$  from the observed nonzero rainfall values.  
 160 5. Generate a Gaussian time series with the desired ACS: Simulate a Gaussian time series  $z(t)$  using an AR  
 161 model of sufficiently large order to reproduce  $\rho_Z(\tau)$ .  
 162 6. Transform the Gaussian time series into a positive rainfall time series: Convert the Gaussian series to a  
 163 rainfall series by applying  $x(t) = Q_X(\Phi_Z(z(t)))$  where  $Q_X$  is the quantile function of the fitted rainfall  
 164 distribution and  $\Phi_Z$  is the standard Gaussian CDF.

165 This approach ensures that the generated rainfall magnitudes maintain the observed probability distribution  
 166 while preserving the Spearman's autocorrelation structure which acts as a proxy in reproducing the dependence  
 167 structure among the wet days.

168 Each process is designed to accurately reproduce key rainfall characteristics: the binary process ensures  
 169 the correct frequency and clustering of wet and dry periods, while the continuous process captures the temporal  
 170 dependence of rainfall magnitudes and preserves the observed distribution. The final rainfall time series is  
 171 generated by multiplying the binary and continuous series, ensuring that rainfall occurs only on wet days and  
 172 follows the observed statistical properties. For further details on the CoSMoS-2s model beyond the rank-based  
 173 configuration, please refer to Papalexioiu (2022).

### 174 2.3. Tuning CoSMoS-2s in Italy's Arno River Basin

175 In the previous section, we outlined the step-by-step procedure for applying the rank-based CoSMoS-2s model.  
 176 However, several key configurations still require careful consideration. Specifically, these include defining  
 177 seasonality, selecting the marginal distribution, choosing between parametric or empirical autocorrelations,  
 178 and determining the order of the autoregressive model. The following choices were primarily guided by the  
 179 need to establish a minimal configuration that minimizes numerical estimations and parameter complexity.  
 180 Additionally, some decisions were informed by the preliminary analysis of rainfall data in the Arno River  
 181 Basin.

182 Regarding seasonality, rainfall patterns typically vary across the year, and it is common to assume that  
 183 statistical properties remain consistent within each season, allowing model parameters to be calibrated  
 184 accordingly. However, determining the optimal number of seasons is not trivial and often involves a degree of  
 185 subjectivity. As noted in Papalexioiu (2022), a conventional approach is to treat each month as a separate season  
 186 and calibrate the model monthly. Given the moderate length of the rainfall records in the Arno River Basin,  
 187 we opted for a simpler and more practical approach by using the four meteorological seasons: Winter  
 188 (December–February), Spring (March–May), Summer (June–August), and Autumn (September–November).  
 189 This strategy reduces model complexity while maintaining accuracy, as aggregating months into broader  
 190 seasons provides more stable parameter estimates when the data supports this generalization.

191 Selecting an appropriate marginal distribution is crucial, as it directly affects both the accuracy and  
 192 parsimony of the model. Several valid options exist, including the Gamma, Weibull, and Generalized Gamma  
 193 distributions, with more complex choices being particularly suitable for finer time scales. While three-  
 194 parameter distributions offer greater flexibility in describing the data, they also increase computational  
 195 complexity. A preliminary analysis of rainfall in the Arno River Basin indicated that the well-known two-  
 196 parameter Gamma distribution provides a good balance between simplicity and accuracy, making it a suitable  
 197 choice. The Gamma distribution, with probability density function

$$f_g(x; \beta, \gamma) = \frac{1}{\beta \Gamma(\gamma)} \left(\frac{x}{\beta}\right)^{\gamma-1} \exp\left(-\frac{x}{\beta}\right) \quad (4)$$

198 and  $\beta$  and  $\gamma$  being positive scale and shape parameters respectively, effectively captures the distribution of  
 199 nonzero rainfall values based on visual and quantitative inspection. Given the chosen seasonal framework, we  
 200 fitted the Gamma distribution separately for each of the four seasons to ensure seasonally consistent  
 201 parameterization. The distribution was fitted numerically by optimizing its parameters to minimize the  
 202 difference between the empirical and theoretical cumulative distribution functions, ensuring an accurate and

203 parsimonious representation of nonzero rainfall values. Other methods, such as method of moments, L-  
204 moments, or maximum likelihood estimation, could also be used (see Nerantzaki and Papalexiou, 2022, for a  
205 detailed review of fitting methods).

206 The last two key modeling choices involve selecting either a parametric or empirical ACS and determining  
207 the order of the AR model. These choices are interrelated, as the strength and structure of the ACS guide the  
208 appropriate AR order. To avoid the complexity of fitting a parametric ACS model, which would require  
209 numerical estimation of additional parameters, we opted for the empirical ACS, offering greater flexibility and  
210 direct alignment with the observed data. Analysis of daily rainfall in the Arno River Basin—consistent with  
211 typical daily rainfall behavior—revealed weak autocorrelation beyond short lags. Both the binary and  
212 continuous processes showed that a two-lag autocorrelation structure sufficiently captures rainfall persistence.  
213 Consequently, we set the AR model order to  $p = 2$ , ensuring it effectively represents short-term dependencies  
214 without overfitting.

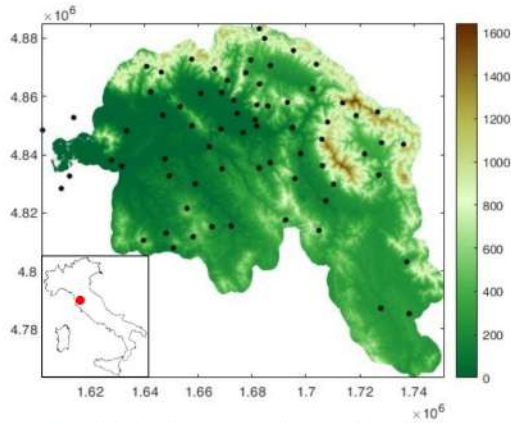
### 215 3. Operational Application in the Arno River Basin

216 This section presents the performance evaluation of CoSMoS-2s. Section 3.1 introduces the study area, dataset,  
217 and data selection process. Section 3.2 examines the model’s performance at a single rain gauge, while Section  
218 3.3 extends the analysis to the entire dataset.

#### 219 3.1. Study Area and Data Selection

220 To evaluate the performance of the CoSMoS-2s rainfall simulation model, we applied it to real-case studies  
221 using data from the Arno River Basin’s rain gauge network. The basin, covering approximately 8,230 km<sup>2</sup>, lies  
222 primarily within the Tuscany region in Central Italy and spans the Northern Apennine chain. Its topography  
223 transitions from mountains to rolling hills and flatlands, eventually reaching the Tyrrhenian Sea. The region  
224 receives an average annual precipitation of about 900 mm, exhibiting significant sub-annual variability. The  
225 basin is densely populated, with major urban centers including Florence and Pisa.

226 The Arno River Basin rain gauge network consists of 123 stations, with a recently compiled dataset  
227 serving as a benchmark for various rainfall modeling studies beyond this work. To ensure the availability of  
228 sub-hourly data—critical for evaluating high-resolution rainfall models—we focused on a 20-year period  
229 (2001–2020) and initially selected 81 stations with 15-minute resolution and less than 10% missing data per  
230 year. Although this study focuses on daily rainfall simulations, CoSMoS-2s outputs will be used in a separate  
231 study to drive a disaggregation model for finer-scale rainfall generation. Therefore, selecting stations with 15-  
232 minute resolution ensures the dataset remains suitable for future assessments.



233 **Figure 1:** Case study data set - 70 rainfall time series observed in rain gauges (black dots) located the Arno  
234 River Basin, Italy.  
235

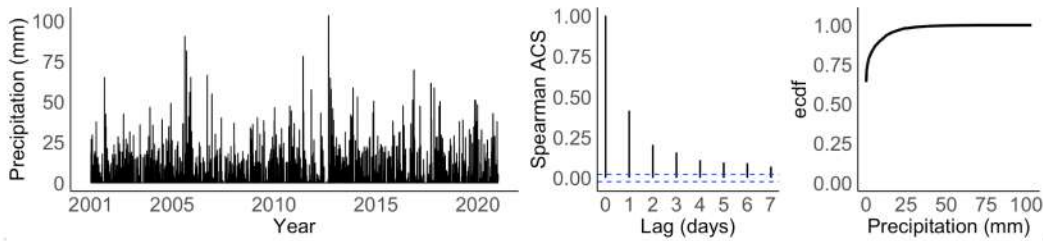
236 Additionally, a preliminary analysis was conducted to assess data quality and characterize rainfall patterns  
237 at each location. This included: (1) dry spell frequencies at daily and 15-minute timescales, (2) maximum  
238 recorded rainfall values at the same timescales, and (3) autocorrelation structures at a daily resolution for lags  
239 of 1, 2, and 3 days. These checks identified two stations with unrealistic correlation structures and extreme  
240 values, leading to their exclusion. Additional filtering was necessary to ensure suitability for disaggregation

241 model applications, as certain stations had anomalous parameter values due to limited sample sizes (see  
 242 Cappelli et al., 2025; under review). After applying these criteria, 70 rainfall time series were retained for  
 243 analysis (Figure 1 shows their spatial distribution).

244 The case study analysis begins with a detailed evaluation of a single rain gauge (Section 3.2), comparing  
 245 observed and simulated time series. It then extends to all selected stations (Section 3.3) to provide a broader  
 246 assessment of model performance.

### 247 3.2. Simulating Rainfall at a Single Location

248 The selected daily time series (Figure 2a) exhibits short-term dependence, as evident from the rapid decline in  
 249 ACS coefficients after lag 1, indicating that precipitation on previous days has little influence on subsequent  
 250 values (Figure 2b). The empirical cumulative distribution function (ecdf) further characterizes the data  
 251 distribution, revealing that approximately 60% of the values are zero, highlighting the probability of zero in  
 252 the observed time series (Figure 2c). Beyond the zero values, most observations are concentrated within a  
 253 relatively narrow range, with cumulative probability increasing gradually from low to high values; for instance,  
 254 75% of the values fall below 0.8.



255 **Figure 2:** Plots of the daily time series for station TOS01001284: (a) observed daily time series, (b) empirical  
 256 Spearman's ACS, and (c) empirical cumulative distribution function (ecdf).  
 257

258 Table 1 provides a detailed summary of the data, categorized by quarterly seasonality. For each season,  
 259 summary statistics are reported for nonzero values.

260 **Table 1:** Summary statistics of the observed daily time series, including sample size, mean, standard deviation,  
 261 minimum, percentiles (25th, 50th, 75th, 90th, 95th, 99th), maximum, the third and fourth L-moments.

Season	<i>n</i>	mean	sd	min	25%	50%	75%	90%	95%	99%	max	L- mom3	L- mom4
Winter	821	6.26	8.73	0.2	0.4	2.4	8.8	17.40	23.60	38.56	54.8	0.47	0.20
Spring	716	6.50	7.65	0.2	0.8	3.8	9.4	17.00	21.95	35.16	47.6	0.39	0.15
Summer	387	6.64	10.01	0.2	0.6	2.8	9.7	17.56	22.14	42.62	90.4	0.48	0.23
Autumn	711	8.82	13.22	0.2	0.4	3.0	11.7	25.60	36.90	58.60	103.2	0.50	0.23

262 The large sample size (*n*) in Winter and Spring seasons enhances the reliability of their statistics compared to  
 263 Summer, which has a smaller sample size. Autumn has the highest mean value (8.82) and the greatest  
 264 dispersion ( $sd = 13.22$ ), followed by summer (mean = 6.64,  $sd = 10.01$ ), while winter has the lowest mean  
 265 (6.26). The maximum values indicate that extreme events are more frequent in Autumn (max = 103.2) and  
 266 Summer (90.4), compared with Winter (54.8) and Spring (47.6). Quantile analysis confirms that Autumn and  
 267 Summer seasons have heavier tails and a higher probability of extreme events, as evidenced by higher L-mom3  
 268 and L-mom4. Winter and Spring, on the other hand, exhibit less skewed distributions, with less variability.

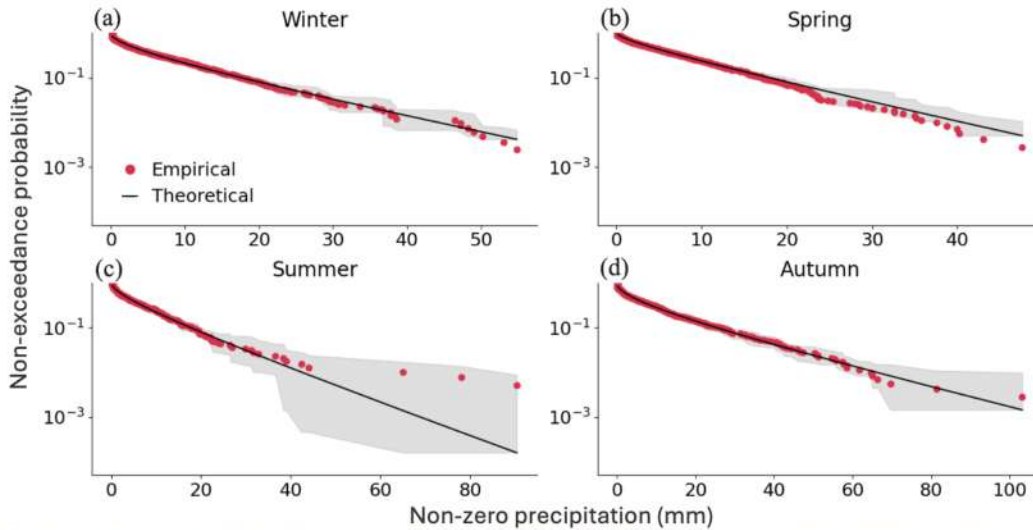
269 The Gamma distribution parameters (Table 2, left side) are estimated by numerically minimizing the  
 270 distance between the empirical and theoretical distribution functions. Winter and Autumn seasons are  
 271 characterized by a more skewed distribution, indicative of rare but intense events, and b) Spring and Summer  
 272 seasons have more symmetrical distributions, suggesting less intense and more frequent precipitation. Finally,  
 273 Autumn is the season with the greatest variability in precipitation, suggesting that it is more prone to extreme  
 274 and unpredictable events.  
 275

276 **Table 2:** Gamma parameter estimates after calibration and the Spearman's ACS coefficients ( $\rho_S(1)$ ,  $\rho_S(2)$   
 277 and  $\rho_S(3)$ ) of the Binary and Continuous time series.

Season	Gamma Parameters		Binary TS			Continuous TS		
	Shape	Scale	$\rho_S(1)$	$\rho_S(2)$	$\rho_S(3)$	$\rho_S(1)$	$\rho_S(2)$	$\rho_S(3)$
Winter	0.47	13.68	0.42	0.18	0.10	0.15	0.06	0.05
Spring	0.64	10.93	0.38	0.19	0.15	0.06	-0.02	0.01
Summer	0.53	12.48	0.26	0.09	0.08	-0.06	-0.08	-0.02
Autumn	0.42	21.56	0.36	0.173	0.09	0.10	-0.01	-0.01

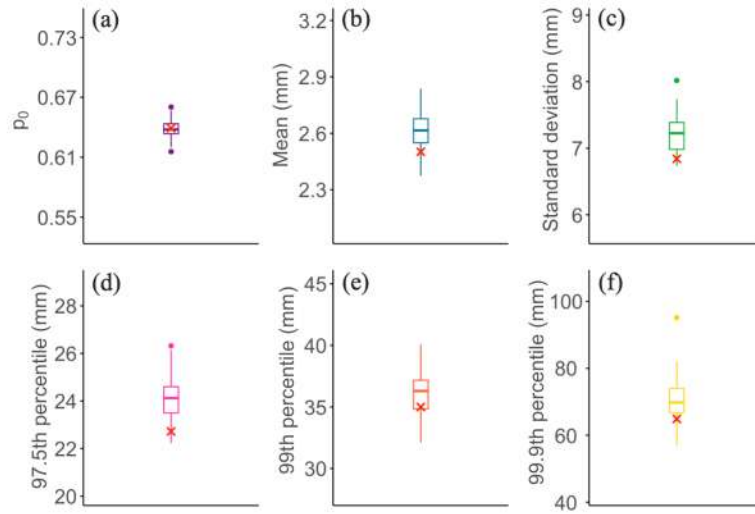
278 Spearman's ACS values of the Binary and Continuous (Table 2, right side) across seasons reveal distinct tem  
 279 poral dependence patterns. For the binary time series, correlation is highest at lag 1 ( $\rho_S(1)$ ) and decreases wit  
 280 h increasing lag. Winter season shows the strongest dependence ( $\rho_S(1) = 0.42$ ), followed by Spring (0.38), A  
 281 utumn (0.36), and Summer (0.26), indicating greater persistence of wet states in colder seasons. For the conti  
 282 nuous time series, correlation is generally lower, suggesting weaker dependence in rainfall intensity compare  
 283 d to wet/dry states. Winter and Autumn seasons show moderate positive correlations ( $\rho_S(1) = 0.15, 0.10$ ), wh  
 284 ile Spring and Summer exhibit weaker or even negative values ( $\rho_S(1) = -0.06$  in Summer).

285 To evaluate the adequacy of the selected Gamma distribution in describing the data, we provide a visual  
 286 comparison of the exceedance probabilities obtained from the fitted parametric distribution for nonzero rainfall  
 287 values with those derived from the empirical distribution providing the 95% confidence interval (gray region)  
 288 of the Gamma distribution (Figure 3). For the four seasons, the results show a good agreement, confirming the  
 289 Gamma distribution's effectiveness in accurately representing non-zero values. The Gamma distribution  
 290 provides a good approximation of the distribution of nonzero values, although some discrepancies emerge,  
 291 particularly for higher values of precipitation. The Winter (Figure 3a) and Spring (Figure 3b) seasons show  
 292 similar behavior, with good agreement between the empirical and theoretical distributions. Summer (Figure  
 293 3c) and Autumn (Figure 3d), on the other hand, show greater dispersion in the highest values of precipitation.  
 294 In particular, Summer has the widest confidence interval, suggesting that the Gamma distribution may not  
 295 adequately capture the tail of the distribution.  
 296



297 **Figure 3:** Probability plots showing the Gamma fitted distribution to nonzero rainfall, its 95% confidence  
 298 interval (gray region), and the empirical distribution for the four seasons: (a) Winter, (b) Spring, (c) Summer,  
 299 and (d) Autumn.  
 300

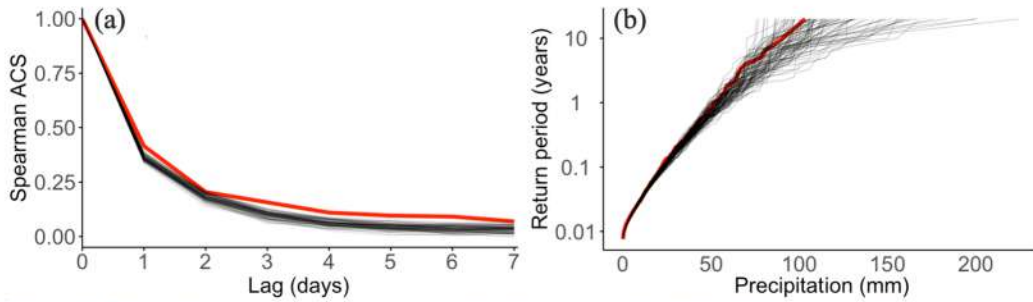
301 Following calibration, we then evaluate the model's performance by comparing several statistical and  
 302 hydrological properties estimated from the entire observed time series with those of 100 simulated daily time  
 303 series, without accounting for the quarterly seasonality. We start analyzing the comparison of the probability  
 304 of zero ( $p_0$ ) and summary statistics (mean, standard deviation, 97.5th, 99th, and 99.9th percentiles) of the 100  
 305 simulated daily time series, represented by boxplots, alongside the observed daily time series, which is marked  
 306 with red crosses (Figure 4). Note that the red crosses lie within the boxplots indicating that the simulated time  
 307 series closely replicate the characteristics of the observed time series. Moreover, the simulated values align  
 308 well with the central tendency and dispersion of the observed data, as reflected by the proximity of the red  
 309 crosses to the median and interquartile ranges of the boxplots. In addition, the red crosses fall within the range  
 310 of the simulated extreme percentiles (Figure 4e and 4f), suggesting that the simulations successfully capture  
 311 not only the average behavior but also the variability and rare events present in the observed data.



312

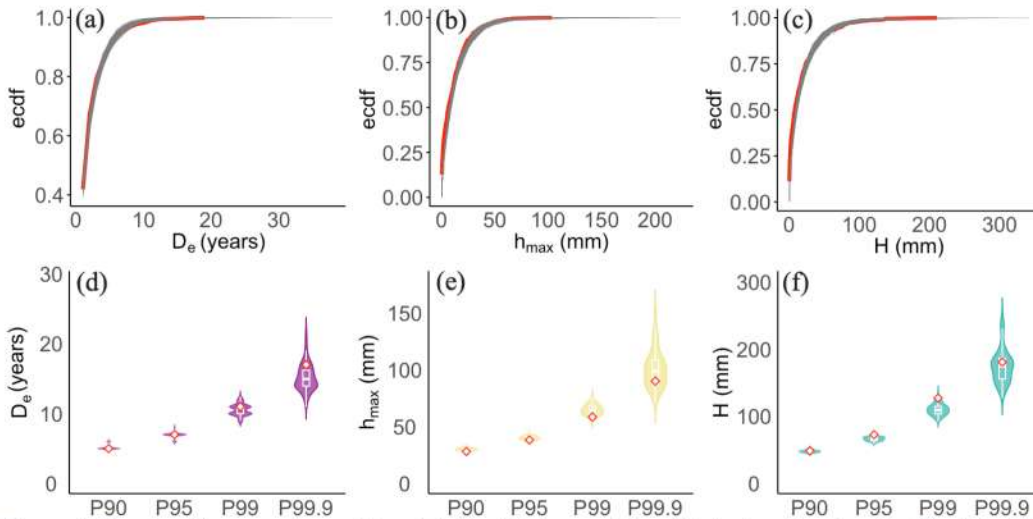
313 **Figure 4:** Plots of the comparison between the summary statistics of the nonzero values of the observed daily  
 314 time (red cross) with those of the 100 simulated (boxplots) daily time series: (a) probability of zero  $p_0$ , (b)  
 315 Mean, (c) Standard deviation, (d) 97.5th percentile, (e) 99th percentile and (f) 99.9th percentile.

316 We evaluate the performance of the minimalistic CoSMoS-2s setting applied here by assessing its ability to  
 317 accurately reproduce the monthly seasonal components (Figure 5) of the observed daily time series within  
 318 the 100 simulated daily time series. The objective is to compare the monthly mean values derived from the  
 319 observed time series (depicted as red diamonds) with the distribution of monthly mean precipitation values  
 320 resulting from the simulated time series (represented by violin plots). Within each violin plot, the white  
 321 boxplot highlights the quartiles and median. Fall months (September-November) exhibit greater variability in  
 322 the simulations compared to the summer months (June-August). In some months (e.g., April, May, October),  
 323 the observed values align closely with the median of the simulated data. However, in other months (e.g.,  
 324 January, September, November), discrepancies arise, with the model tending to either overestimate or  
 325 underestimate the mean monthly precipitation. This discrepancy could be attributed to the choice of using a  
 326 quarterly seasonality approach or to the limited ability of the marginal distribution to fully capture the  
 327 characteristics of the data. We also clarify that the distribution was fitted using the least squares method,  
 328 which does not explicitly preserve the mean and standard deviation of the data. Therefore, any slight  
 329 deviations in these statistics are not limitations of the model itself, but rather a consequence of the fitting  
 330 approach. If the method of moments had been used instead, the fitted gamma distribution would have  
 331 explicitly matched the observed mean and standard deviation. However, we opted for the least squares  
 332 method because it offers a better overall fit across the entire distribution, particularly in capturing the shape  
 333 and tail behavior more accurately.



358  
359 **Figure 7:** Plots of the comparison between (a) the Spearman ACS of the 100 simulated (black lines) and  
360 observed (red line) daily time series, (b) the daily rainfall accumulation as a function of the return period based  
361 on the 100 simulated (black lines) and observed (red line) time series.

362 Finally, the model's performance is evaluated by analyzing the rainfall events. For this analysis, events  
363 are selected based on a rule that requires a minimum 24-hour dry period between two consecutive and  
364 independent events. This allows us to focus on key characteristics of the rainfall events, such as event duration  
365 ( $D_e$ ), total precipitation amount ( $H$ ), and peak intensity ( $h_{max}$ ). After identifying events in both the observed and  
366 simulated time series, we compare the empirical cumulative distribution functions of duration (Figure 8a),  
367 total precipitation (Figure 8b), and peak intensity (Figure 8c). Overall, the simulated and observed values for  
368 durations, total amounts, and peaks show good agreement. However, some differences emerge in the tails of  
369 the distributions, indicating discrepancies between the simulated and observed values for extreme events. To  
370 provide a comprehensive overview of the proposed analysis, we also compare the 90th, 95th, 99th, and 99.9th  
371 percentiles of event duration (Figure 8d), total event precipitation (Figure 8e), and peak event (Figure 8f). The  
372 results suggest that the CoSMoS-2s model effectively reproduces the event-scale characteristics of the  
373 observed daily time series. Although there is increased variability at the higher percentiles (Figure 8d, 8e, 8f),  
374 the agreement between the observed values and the simulated ones remains strong, indicating that the model  
375 can capture the behavior of extreme events with reasonable accuracy.

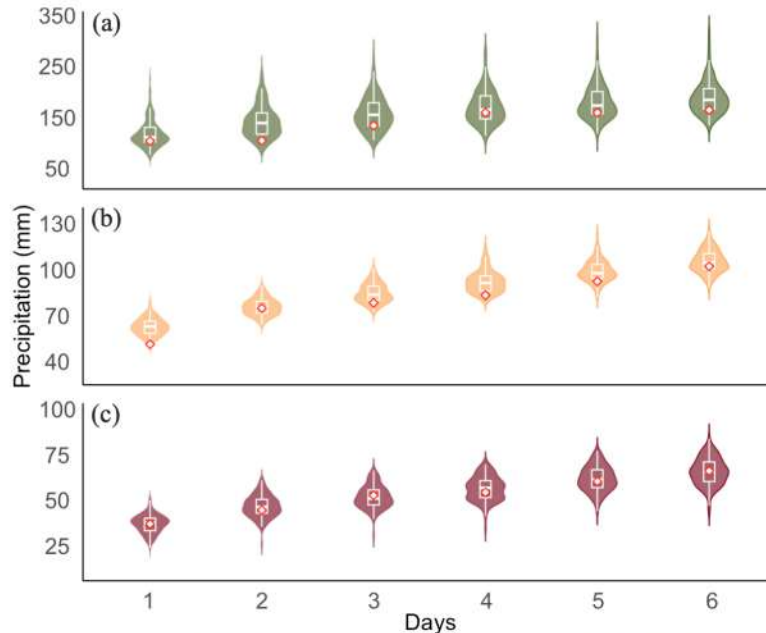


376  
377 **Figure 8:** Event-scale comparisons plots of 1) Empirical cumulative distribution functions (ecdf) of the 100  
378 simulated daily time series (black lines) and that of the observed (red line) daily time series for: (a) the duration  
379  $D_e$ , (b) total precipitation  $H$  and (c) peaks  $h_{max}$ ; 2) 90th, 95th, 99th and 99.9th (P90, P95, P99, P99.9)  
380 percentiles of the 100 simulated (violin plots) and those of observed (red diamonds) daily time series for: (d)  
381 the duration  $D_e$ , (e) total precipitation  $H$  and (f) peaks  $h_{max}$ .



334  
335 **Figure 5:** Comparison between monthly seasonal components of observed (red diamond  $\diamond$ ) and those of the  
336 100 simulated (violin plots) daily time series

337 In Figure 6, the comparison of the highest (1st), median (10th), and lowest (20th) ranked annual maximum  
338 rainfall depths across various durations (1 to 6 days), computed from 100 simulated daily time series (violin  
339 plots) and the observed record (red diamonds), reveals that the observed annual maximum rainfall values fall  
340 within the range of the simulated extremes for all considered durations. However, for the highest  
341 ranked annual maximum rainfall depths (Figure 6a), greater variability is observed across different durations.  
342 Overall, the simulated daily time series can be considered to adequately replicate the ranked annual maximum  
343 rainfall depths for different daily durations when compared to the observed series.



344  
345 **Figure 6:** Plots of the comparison of: (a) the highest, (b) the median, and (c) the lowest observed ranked annual  
346 maxima at different durations (red diamonds) and the annual maxima extracted from the 100 simulated daily  
347 time series (violin plots).

348 Generally, it is essential for a rainfall simulation model to accurately replicate both the autocorrelation  
349 structure of the observed time series and the rainfall to ensure reliable predictions. The autocorrelation  
350 structures of the 100 simulated daily time series (Figure 7a) closely resemble that estimated from the observed  
351 data, indicating the ability of CoSMoS-2s model to accurately reproduce the Spearman's ACS. Furthermore,  
352 the comparison of the daily rainfall accumulation as a function of the return period based on the 100 simulated  
353 and observed time (Figure 7b), shows that the results are tightly clustered for lower return periods,  
354 demonstrating good agreement with the observed relationship. However, as the return period increases, so does  
355 the variability of the simulated rainfall amounts, which is a common feature in extreme value analysis. This  
356 increased dispersion at higher return periods reflects the inherent uncertainty of rarer events, indicating that  
357 while the model can capture the overall trend, its accuracy diminishes for extreme and infrequent events.

382 Based on the comprehensive comparisons between the simulated and observed daily time series, we can  
383 conclude that the model effectively captures the key statistical properties of daily precipitation data with a high  
384 degree of accuracy for the considered raingauge. Furthermore, despite the limited sample size of the observed  
385 data, the model successfully reproduces the statistical properties of the rainfall time series.

### 386 3.3. Model Performance Across the Entire Dataset

387 In this section, the previous analyses are extended to the entire Arno River dataset to evaluate the model's  
388 capability in reproducing key statistical properties of the observed daily time series. To provide a clear view  
389 of the model's performance, we select 24 indicators because they comprehensively capture different aspects of  
390 the statistical properties of the simulated time series. These indicators help evaluate the model's ability to  
391 replicate real-world precipitation patterns at various temporal scales, which is crucial for accurate simulations.  
392 The three groups of indicators are defined as follows:

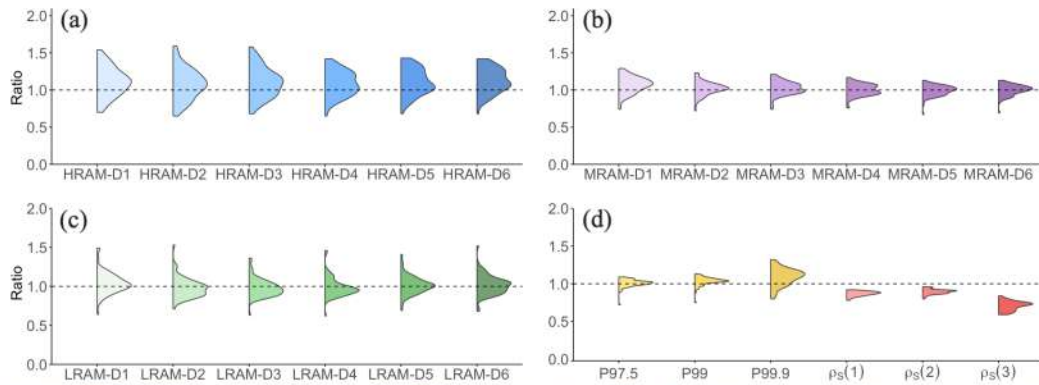
- 393 1. Annual Maxima indicators: 18 indicators related to the 1st (highest), the 10th (median), and the 20th  
394 (lowest) ranked annual maxima rainfall intensities at the different time scale extracted from the  
395 corresponding 100 simulated time series, named as HRAM-Dn (Highest Ranked Annual Maxima with  
396 duration n); MRAM-Dn (Median Ranked Annual Maxima with duration n); and LRAM-Dn (Lowest  
397 Ranked Annual Maxima with duration n) where n ranges from 1 to 6 days.
- 398 2. Percentile indicators: three indicators related to the three percentiles (97.5th, 99th, and 99.9th), named  
399 P97.5, P99, and P99.9.
- 400 3. Spearman's ACS indicators: three indicators related to the Spearman's ACS coefficients for lags equal to  
401 1, 2, and 3, denoted as  $\rho_S(1)$ ,  $\rho_S(2)$  and  $\rho_S(3)$ .

402 Each indicator is defined as the ratio between the median of the 100 values resulting from the simulated time  
403 series and the value calculated from the observed time series. Choosing the median helps ensure that the  
404 analysis is not influenced by outliers, leading to more robust and reliable results. The ratio indicates whether  
405 the model tends to overestimate or underestimate a specific statistical property under consideration.  
406 Specifically,

- 407 1. A ratio close to 1 indicates that the median is near the observed value, indicating good reproduction of a  
408 specific property
- 409 2. A ratio greater than 1 suggests a tendency of the model to overestimate.
- 410 3. A ratio less than 1 indicates a tendency of the model to underestimate.

411 The performance of the CoSMoS-2s model is evaluated by analyzing the behavior of the proposed indicators  
412 calculated for the entire Arno River Basin dataset. The indicator distributions (Figure 9) resulting from the  
413 analysis provide a clear understanding of the model performance. The distribution of HRAM-Dn indicators  
414 (Figure 9a) exhibits greater variability compared to the distributions of MRAM-Dn and LRAM-Dn indicators  
415 (Figure 9b and 9c), as expected. The latter two are characterized by lower variability and their distributions  
416 are centered around 1, indicating the model's ability to reproduce these properties accurately. The HRAM-Dn  
417 indicators suggest that the model slightly overestimates the highest ranked annual maxima rainfall intensities  
418 at different durations. The percentile indicators (represented by gold distributions in Figure 9d) show that  
419 CoSMoS-2s is generally perform well in reproducing the extreme values observed in the daily time series.  
420 However, for the 99.9th percentile, there is a tendency for the model to overestimate the extreme tail of the  
421 distribution. Finally, the Spearman's ACS coefficient indicators (orange distributions in Figure 9d) show that  
422 the model underestimates the Spearman's ACS coefficients, with the underestimation becoming more  
423 pronounced as the time lag increases. This matches the findings in Figure 7a, which pertain to a single  
424 raingauge. The results highlight the model's ability to closely mimic intermittent processes, despite some  
425 minor over- or underestimations in some statistical properties. These discrepancies could potentially be  
426 mitigated by using longer time series, which would provide more information and allow for greater accuracy.

427



428 **Figure 9:** Plots of the CoSMoS-2s performance evaluation across the 70 daily time series based on three  
 429 criteria: (a) the 1st (highest), (b) 10th (median), and (c) 20th (lowest) ranked annual maxima at six durations  
 430 extracted from 100 simulated time series, denoted as HRAM-Dn (Highest Ranked Annual Maxima for duration  
 431 n), MRAM-Dn (Median Ranked Annual Maxima for duration n), and LRAM-Dn (Lowest Ranked Annual  
 432 Maxima for duration n), where n ranges from 1 to 6 days; (d) the 97.5th, 99th, and 99.9th percentiles of the  
 433 rainfall distribution (P97.5, P99, and P99.9) and the Spearman's ACS coefficients at lag 1, 2 and 3  
 434 ( $\rho_S(1)$ ,  $\rho_S(2)$  and  $\rho_S(3)$ ).  
 435

436 **5. Conclusion**

437 In this study, we conduct a comprehensive evaluation of the two-state intermittent model (CoSMoS-2s) applied  
 438 for single-site time series generation, as introduced by Papalexiou (2022), using real-world case studies. Our  
 439 aim is to bridge the model's inherent flexibility with its practical usability by assessing its performance in real-  
 440 world scenarios, ensuring that it remains both theoretically robust and operationally applicable, even under  
 441 data-limited conditions. We used a single daily time series from a raingauge in the Arno River Basin and the  
 442 entire Arno River dataset, comprising 70 daily time series. In addition to the inherent challenges of working  
 443 with real data, a significant difficulty arises from the limited data availability, as each time series covers only  
 444 a 20-year period. To address this, we implemented a data grouping based on quarterly seasonality, rather than  
 445 the monthly scale approach suggested in Papalexiou (2022).

446 To assess the model's performance, we generated 100 daily time series using the observed daily time  
 447 series and compared various statistical properties, including summary statistics, Spearman autocorrelation  
 448 structure, monthly seasonal components, as well as the highest, median, and lowest ranked annual maxima  
 449 rainfall intensities at different durations. At the event scale, properties such as event duration, total event  
 450 amount, and event peak were also analyzed. Crucially, our study aimed to configure CoSMoS-2s in a  
 451 parsimonious manner, ensuring easy calibration while preserving strong performance—an essential factor for  
 452 operational applications. Across the case studies, the model demonstrated a remarkable ability to reproduce an  
 453 intermittent process while preserving statistical characteristics. Furthermore, the implementation was  
 454 computationally efficient due to the choice of quarterly seasonality and the use of empirical Spearman's ACS  
 455 over a parametric one. By employing a Gamma distribution (2 parameters) and considering four seasons, the  
 456 model was streamlined to only eight parameters. This minimalistic setup not only reduces computational  
 457 burden but also enhances the model's applicability in practical hydrological studies, particularly in data-scarce  
 458 regions.

459 Overall, these results highlight that CoSMoS-2s is not only a theoretically flexible model but also a  
 460 practical and robust tool for rainfall simulation under real-world constraints. By validating its performance in  
 461 an operationally relevant setting, this study reinforces its potential for use in hydrological applications where  
 462 data availability and calibration effort are limiting factors.

463

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467

468 **Authors' contributions** Conceptualization: all authors; Methodology: SM.P.; Software and Validation: F.C.;  
469 Formal analysis and Investigation: all authors; Data Curation: F.C.; Writing (original draft): F.C., SM.P.;  
470 Writing (review & editing): all authors; Visualization and Supervision: all authors; Funding Acquisition: E.V.,  
471 S.G.

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475 **Data availability** Datasets were generated or analysed during the current study will be made available on  
476 request.

477 **Code availability** Not applicable.

#### 479 **Declarations**

480 **Ethics approval** This work complied with all the necessary ethical approval processes.

481 **Competing interest** The authors declare no competing interests.

482 **Consent to participate** Not applicable.

483 **Consent for publication** The author agrees to publish.

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## **5. Multisite Rainfall Simulation in the Arno River Basin in Italy**

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# Multisite Rainfall Simulation in the Arno River Basin in Italy

## Summary

This document presents a comprehensive multisite precipitation simulation framework based on the two-state CoSMoS-2s model, designed to realistically reproduce the spatiotemporal characteristics of rainfall across a large domain. Building on the limitations of earlier stochastic rainfall models, CoSMoS-2s explicitly separates the simulation into a binary process for wet/dry states and a continuous process for rainfall intensities, each modeled using Gaussian vector autoregressive (VAR) processes. A semiparametric strategy was employed to combine empirical and parametric components, balancing accuracy and computational feasibility across 70 stations. The binary and nonzero components were calibrated using seasonally adaptive statistics, transformed via correlation transformation functions, and simulated independently before being recombined to produce final time series. The model was evaluated through detailed station-level diagnostics and additional assessments at the multi-station and catchment scale, including comparisons of autocorrelations, distributional moments, wet/dry spell statistics, and scaling properties. Results showed excellent agreement between observed and simulated rainfall characteristics at all spatial scales. Notably, cross-correlations, L-moments, and spell durations were well preserved, with only a minor underestimation of the standard deviation in aggregated catchment precipitation—likely due to spatial averaging. Overall, the CoSMoS-2s framework demonstrated robust performance, capturing key features of rainfall variability, intermittency, and dependence structure over both time and space.

## 1. Methodology and Model Overview

Rainfall is characterized by intermittent and highly variable behavior across different temporal and spatial scales, which makes its simulation inherently challenging. Numerous stochastic models have been proposed over recent decades, including cluster-based point processes (e.g., Neyman-Scott and Bartlett-Lewis), Markov chains for wet/dry transitions, multifractal approaches leveraging scale-invariance, and autoregressive or copula-based frameworks to model temporal and spatial dependence. Despite their utility, none of these methods individually reproduces all essential features of rainfall processes comprehensively.

To overcome these limitations, the Complete Stochastic Modelling Solution (CoSMoS) framework was developed, which combines flexible marginal distributions with transformed Gaussian processes to simultaneously reproduce rainfall intermittency, marginal behavior, and correlation structures. Within this framework, two main variants exist: the single-state CoSMoS-1s (Papalexiou, 2018) and the two-state CoSMoS-2s model (Papalexiou, 2022). The CoSMoS-1s treats rainfall as a single intermittent process using mixed-type marginals and autocorrelation structures, whereas the CoSMoS-2s separates rainfall simulation into two distinct processes—a binary wet/dry process and a continuous rainfall intensity process—allowing explicit preservation of each component's autocorrelation structure and marginal distributions.

Recent analyses (Papalexiou, 2022) have demonstrated that CoSMoS-2s outperforms CoSMoS-1s in realistically reproducing key rainfall characteristics, such as wet and dry spell distributions, moment scaling, and extremes across multiple temporal scales. Its superior performance is attributed to the two-state modeling approach, explicitly capturing rainfall intermittency and intensity dynamics separately.

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Multisite precipitation simulation introduces additional complexity, particularly when applied over large spatial domains. Despite advances in remote sensing technologies and availability of high-resolution datasets, such data typically have limited temporal coverage, necessitating stochastic modeling for generating realistic long-term scenarios. Traditional modeling approaches (point processes, resampling, and disaggregation) often face scalability and accuracy issues when extended spatially, especially regarding extreme events and wet/dry spells. Although parametric time series approaches provide scalability, they remain computationally challenging and are rarely applied to extensive station networks.

In a previous study (Papalexiou et al., 2023), the CoSMoS-1s framework was adapted for large-domain multisite simulation by employing a transformation strategy. To address the computational burden associated with estimating correlation transformation functions (CTFs) for each pair of stations, the approach transformed all time series to mixed-type Uniform distributions. This allowed the use of a single, site-independent CTF, significantly improving computational efficiency. However, since this method inherits the limitations of the CoSMoS-1s approach, the present study instead utilizes a multisite extension of the CoSMoS-2s framework, thereby leveraging its demonstrated advantages in accurately simulating rainfall intermittency and dependence structures across large spatial scales.

## 2. Challenges and Strategies for Multisite Simulation

Multisite rainfall simulation involves capturing complex dependencies both spatially and temporally, dealing with heterogeneity in marginal distributions across locations, and managing computational burdens for large-scale scenarios. As highlighted by Papalexiou et al. (2023), three general modeling strategies, that is, nonparametric, parametric, and semiparametric, can address these challenges, each with distinct strengths and limitations.

- Nonparametric strategies aim to directly reproduce observed spatial cross-correlations and site-specific autocorrelations using empirical estimates derived from transformed time series. This method offers maximum flexibility and accuracy in replicating observed dependencies but becomes computationally intensive and often impractical at large scales due to the complexity involved in estimating and inverting dense correlation matrices.
- Parametric strategies provide a scalable alternative by fitting analytical spatiotemporal correlation structures (STCS). These structures describe correlations as mathematical functions of spatial distance and temporal lag, significantly reducing computational complexity. For example, Papalexiou and Serinaldi (2020) introduced the Clayton-Weibull STCS, a compact model suitable for large-domain simulation. However, parametric models, while efficient, might fail to capture detailed local dependencies accurately due to their parsimonious nature.
- Semiparametric strategies blend empirical and parametric components, using observed correlations at specific temporal lags or spatial scales and parametric structures elsewhere. This hybrid approach combines computational tractability with local realism and accuracy, effectively addressing spatial heterogeneity and nonstationarities. Nevertheless, preserving exact lag-0 correlations is impossible when station-specific autocorrelation structures vary significantly. Papalexiou et al. (2023) demonstrated that this limitation is generally minor if station autocorrelations are reasonably similar.

This study applies the semiparametric approach within the CoSMoS-2s framework, balancing accuracy and computational feasibility to effectively simulate rainfall across multiple sites.

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### 3. Application and Implementation Details

In the multisite CoSMoS-2s implementation, rainfall simulation involves two coupled but distinct processes: (1) a binary process for wet/dry occurrences and (2) a continuous process for rainfall intensity on wet days. Each component is modeled separately using Gaussian parent vector autoregressive (VAR) processes.

#### 3.1 Simulation of the Binary (Wet/Dry) Process

The binary simulation involves the following steps:

1. Transformation to binary series: Original rainfall records at each of the 70 stations are converted into binary sequences (wet days assigned as 1, dry days as 0).
2. Cross-correlations: Empirical lag-0 cross-correlation matrices are estimated monthly across all stations.
3. Autocorrelation structures (ACS): Station-specific empirical autocorrelations are computed separately for each month.
4. ACS parameterization: A parametric Weibull autocorrelation structure is fitted to these empirical ACS values for each station.
5. Correlation transformation: Both empirical lag-0 correlation matrices and the parametric ACS estimates are transformed via a binary correlation transformation function (CTF) to their Gaussian equivalents.
6. Gaussian VAR model: A VAR(10) model with diagonal parameter matrices is utilized, reproducing spatial lag-0 cross-correlations and temporal autocorrelation structures at each station explicitly up to ten days.
7. Probability of zero: Daily probabilities of zero rainfall at each station are estimated using a two-month moving window based on observed data.
8. Binary outcome generation: Gaussian simulations are converted into wet/dry states by applying the daily zero-probability thresholds derived previously in the Gaussian time series.

#### 3.2 Simulation of the Nonzero (Rainfall Intensity) Process

The nonzero rainfall intensities are simulated with a similar but specialized approach:

1. Estimation of parameters: Daily mean and standard deviation of nonzero rainfall at each station are computed using a two-month moving window.
2. Marginal distributions: Gamma distributions parameterized by the seasonal patterns of the mean and standard deviation are fitted individually for each station and each day.
3. Conditional cross-correlations: Monthly empirical lag-0 cross-correlation matrices are estimated conditionally on days with nonzero rainfall across all stations.
4. Conditional ACS: Empirical conditional autocorrelation structures for nonzero values are estimated for each station and month.
5. Parametric ACS fitting: Parametric Weibull ACS models are fitted to the empirical conditional ACSs for each station.
6. Gaussian correlation transformation: Empirical conditional cross-correlation matrices and parametric ACS estimates are transformed into Gaussian correlation structures via a correlation transformation function (CTF), incorporating the previously fitted Gamma distributions.

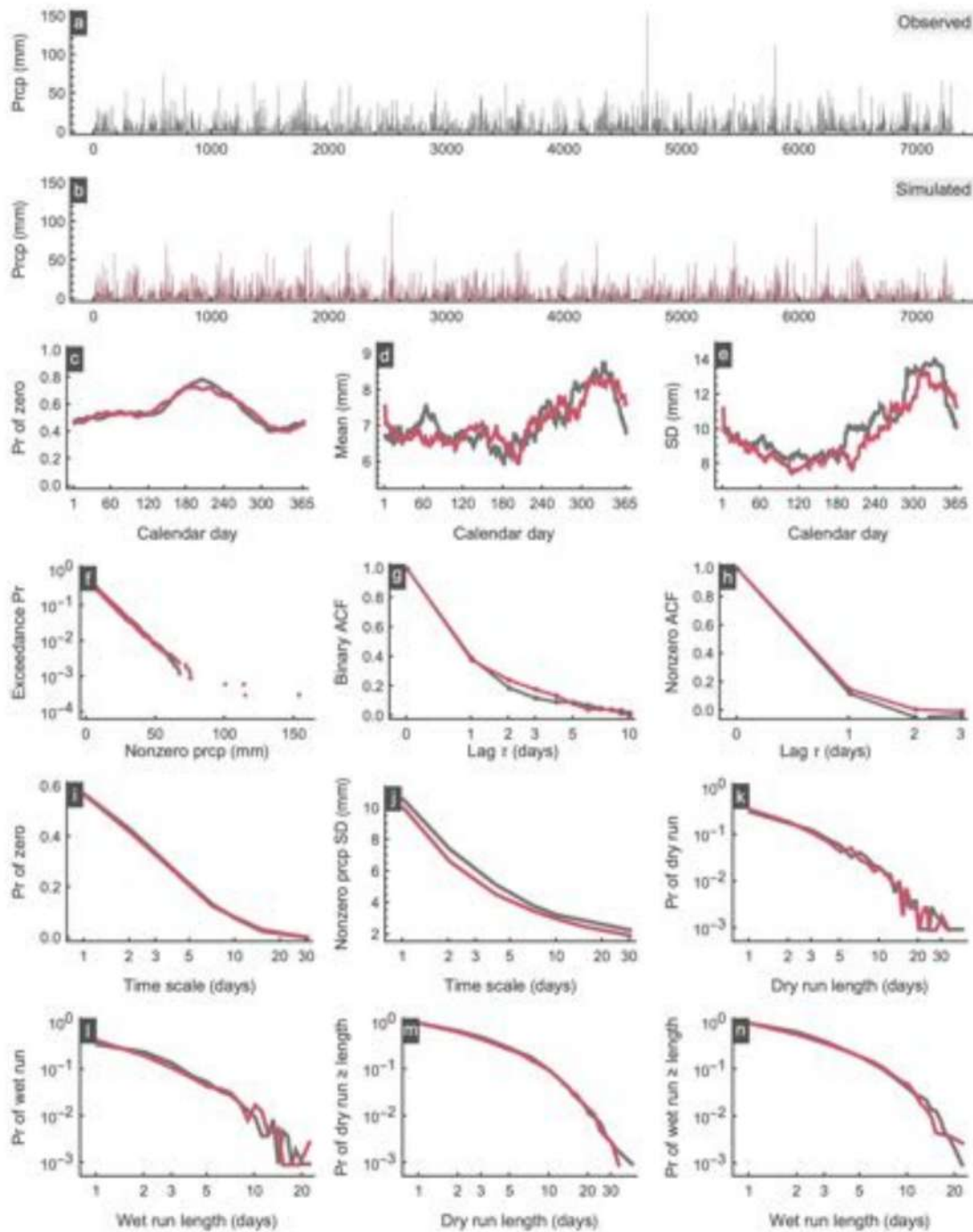
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7. Gaussian VAR simulation: A VAR(1) Gaussian model with diagonal parameter matrices reproduces the spatial cross-correlations and temporal autocorrelations for each station's nonzero rainfall series.
  8. Intensity generation: The simulated Gaussian series are transformed back into rainfall intensities using the inverse of the fitted Gamma distributions on a daily basis.

Finally, the multisite precipitation series are generated by multiplying the binary wet/dry simulations by the nonzero rainfall intensity simulations, producing realistic synthetic rainfall records that preserve key statistical properties at all stations. Further methodological and technical details are discussed comprehensively in Papalexioiu (2018, 2022) and Papalexioiu et al. (2023).

## **4. Results**

### **4.1 Simulation Assessment per Station**

In this study, synthetic precipitation time series spanning 2000 years were generated for each station, provided as four segments of 500 years each. Each station's simulated series was assessed thoroughly by comparing simulated results against the 20-year observed precipitation characteristics. Specifically, a detailed multi-panel graphical summary was created per station, comprising multiple diagnostic plots to evaluate the performance and realism of the simulation comprehensively. Below, we provide the first multi-panel graph for the first station (Pluviometro 00) and next we describe each graph's purpose, construction, and usefulness.



**Figure 1.** Simulation assessment for the first station (Pluviometro 00). (a) and (b) observed and simulated time series; (c) probability of zero rainfall; (d) mean rainfall intensity; (e) standard deviation of rainfall intensity; (f) exceedance probability of nonzero rainfall; (g) binary autocorrelation structure; (h) nonzero rainfall autocorrelation structure; (i) scaling of probability of zero; (j) scaling of standard deviation; (k) probability mass function of dry spell length; (l) probability mass function of wet spell length; (m) complementary cumulative distribution of dry spell length; (n) complementary cumulative distribution of wet spell length.

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- **Panels (a) and (b): Observed and Simulated Rainfall Time Series.** These panels visually depict rainfall daily amount over time, comparing observed precipitation (panel a) and simulated precipitation (panel b). This direct comparison quickly illustrates whether the simulation realistically captures the variability, intensity, and intermittency observed in the actual rainfall record.
  - **Panel (c): Seasonal Probability of Zero Rainfall.** This graph shows the daily probability of zero rainfall (dry days) throughout the year. It compares the seasonal pattern of observed and simulated probabilities, assessing the model's ability to reproduce the annual cycle of intermittency.
  - **Panel (d): Seasonal Mean Rainfall Intensity.** This plot illustrates the average rainfall intensity on wet days for each calendar day, highlighting the seasonal cycle. Comparing observed and simulated values ensures the model accurately represents seasonal patterns of rainfall intensity.
  - **Panel (e): Seasonal Standard Deviation of Rainfall Intensity.** Here, the variability of rainfall on wet days throughout the year is shown. This comparison verifies that the simulation correctly reproduces daily variability and seasonal fluctuations observed historically.
  - **Panel (f): Exceedance Probability of Nonzero Rainfall.** This probabilistic plot provides empirical exceedance probabilities for nonzero daily rainfall amounts, comparing observed and simulated distributions. It ensures the simulated intensities are realistic across the full range, particularly for higher rainfall amounts that are critical for extremes analysis.
  - **Panel (g): Autocorrelation Structure (ACS) of the Binary (Wet/Dry) Process.** This plot shows the autocorrelation of wet/dry sequences, comparing observed and simulated values for different temporal lags (1 to 10 days). It evaluates how well the simulation captures persistence in rainfall occurrence.
  - **Panel (h): Autocorrelation Structure (ACS) of Nonzero Rainfall Intensities.** Similar to the binary ACS, this graph presents the autocorrelation of nonzero rainfall intensities. It helps verify that the simulation realistically preserves the temporal dependence of rainfall amounts.
  - **Panel (i): Scaling of Probability of Zero Rainfall with Time Scale.** This plot examines how the probability of zero rainfall days scales as daily data are aggregated into larger time scales (up to 30 days). It checks whether the simulation maintains realistic intermittency characteristics across multiple temporal resolutions.
  - **Panel (j): Scaling of Standard Deviation for Nonzero Rainfall with Time Scale.** Depicting the standard deviation of nonzero rainfall aggregated over varying time scales, this plot assesses if the simulation maintains realistic variability when transitioning from daily to longer time scales.
  - **Panels (k) and (m): Probability of Dry Spell Length.** These graphs evaluate how well the simulation captures the characteristics of dry spells. Panel (k) shows the probability mass function (PMF)—the probability of observing a dry spell of exactly a specified length, while panel (m) displays the complementary cumulative distribution function (CCDF)—the probability of observing a dry spell of at least the specified length. Together, these plots verify the accurate simulation of dry-spell durations, which are essential for drought risk assessment.

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- **Panel (l) and (n): Probability of Wet Spell Length.** Analogously, these panels assess the durations of wet spells. Panel (l) depicts the probability mass function (PMF), representing the probability of a wet spell lasting exactly a given number of days, whereas panel (n) shows the complementary cumulative distribution function (CCDF), indicating the probability of a wet spell lasting at least that duration. These plots help confirm the model’s effectiveness in reproducing prolonged rainfall events, which are critical for flood management and water resource planning.

All these detailed graphical evaluations for each individual station are provided in the Annex (see Figures A1 to A69). Overall, these results demonstrate that the simulated rainfall series closely match the observed statistical properties, confirming the excellent performance of the adopted modeling approach.

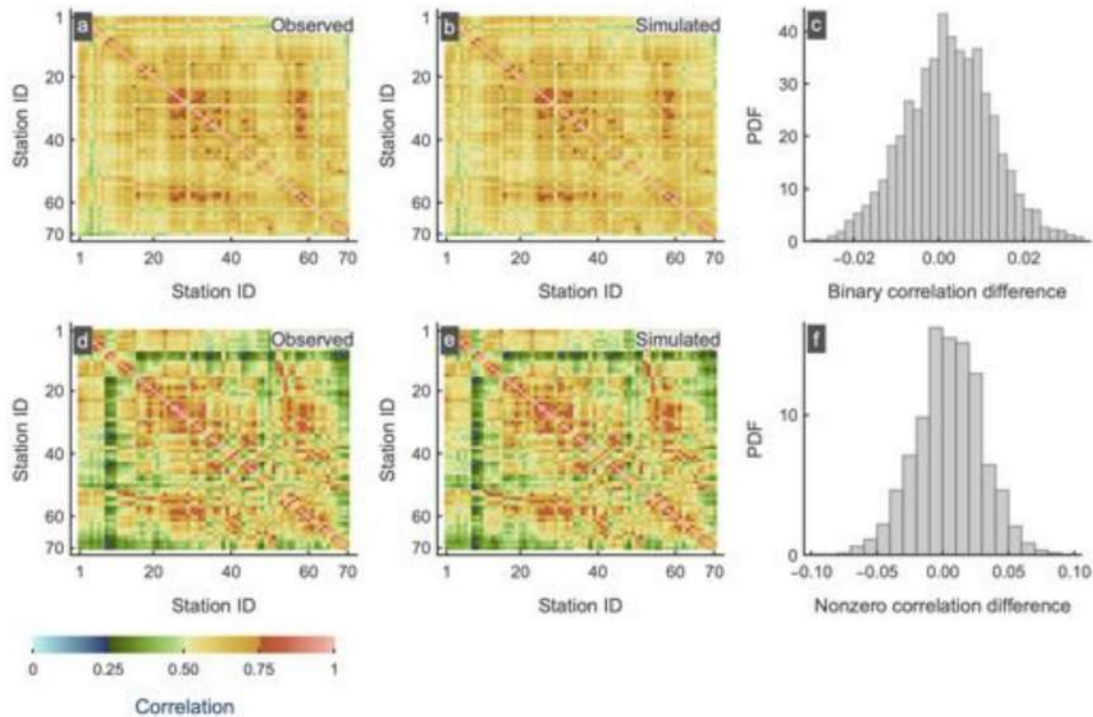
#### 4.2 Simulation Assessment of Multiple Stations and at the Catchment Scale

Beyond the previous detailed analysis conducted for each individual station, additional assessments are provided here to evaluate the performance of the simulation collectively across all stations and at the catchment scale. Specifically, we present side-by-side comparisons of several key statistics across all stations between observed and simulated data, evaluate the lag-0 correlation matrices for both wet/dry (binary) and nonzero rainfall processes, and examine aggregated precipitation at the catchment scale.

These additional analyses are presented through three separate figures, each focusing on a distinct aspect of the simulation assessment. Specifically:

**Observed and simulated lag-0 correlation plot** (Figure 2). The first row (panels a–c) shows the lag-0 correlation matrices for the binary (wet/dry) rainfall process, with the observed correlation matrix (a), the simulated correlation matrix (b), and the histogram of differences between the simulated and observed correlations (c). The second row (panels d–f) provides the same information for the nonzero rainfall process, including observed correlations (d), simulated correlations (e), and the histogram of their differences (f).

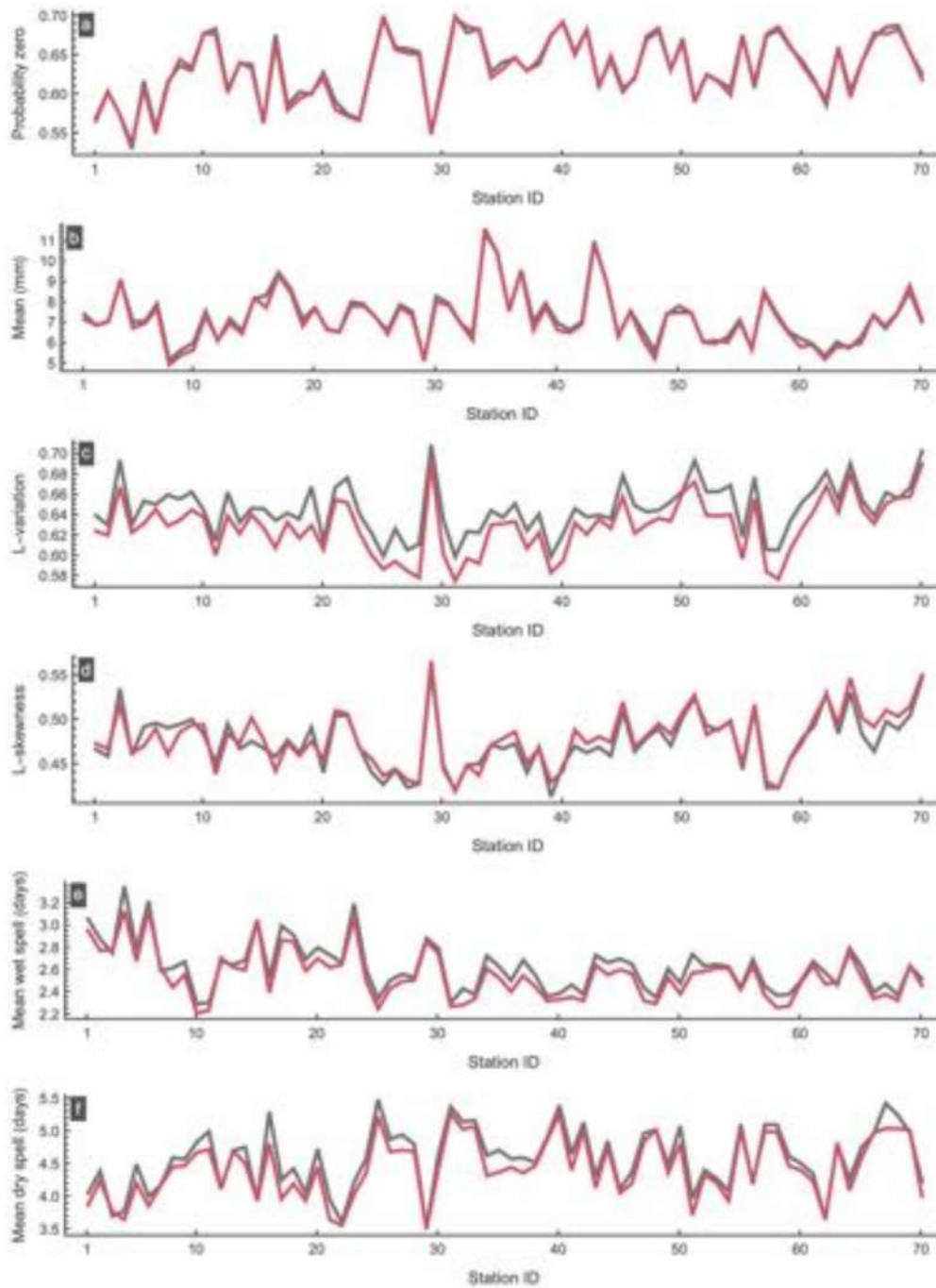
These plots demonstrate the effectiveness of the simulation in capturing spatial dependencies across stations for both rainfall occurrence and intensity. Notably, the color patterns in the correlation matrices are nearly identical between observed and simulated data for both the wet/dry and nonzero processes, indicating strong spatial agreement. This is further confirmed by the histograms of correlation differences, which are tightly centered around zero and show very low deviations, highlighting the high fidelity of the simulated spatial dependence structure. It is also evident that the binary (wet/dry) process exhibits stronger cross-correlations across stations compared to the nonzero rainfall process—reflecting the higher spatial coherence of rainfall occurrence relative to intensity.



**Figure 2.** Cross-correlations among stations. (a–c) Observed and simulated lag-0 correlation matrices for the binary (wet/dry) process and their differences; (d–f) same for the nonzero rainfall process. See text for details.

**Station-level statistics comparison (Figure 3).** This analysis presents observed and simulated statistics computed from the full time series at each of the 70 stations, without seasonal or monthly stratification. The six metrics evaluated are: (a) probability of zero rainfall, (b) mean rainfall intensity, (c) L-variation, (d) L-skewness, (e) mean wet spell length, and (f) mean dry spell length.

The L-variation and L-skewness provide robust, distribution-free measures of variability and asymmetry, respectively, offering a more stable characterization of rainfall properties than traditional moments. Wet and dry spell statistics reflect persistence patterns essential for hydrological applications, including drought and flood modeling. The results indicate that inter-station variability is relatively low, suggesting a hydrologically homogeneous basin. Most importantly, the agreement between observed and simulated statistics is consistently strong across all stations, confirming that the model effectively captures key rainfall characteristics over space and time.



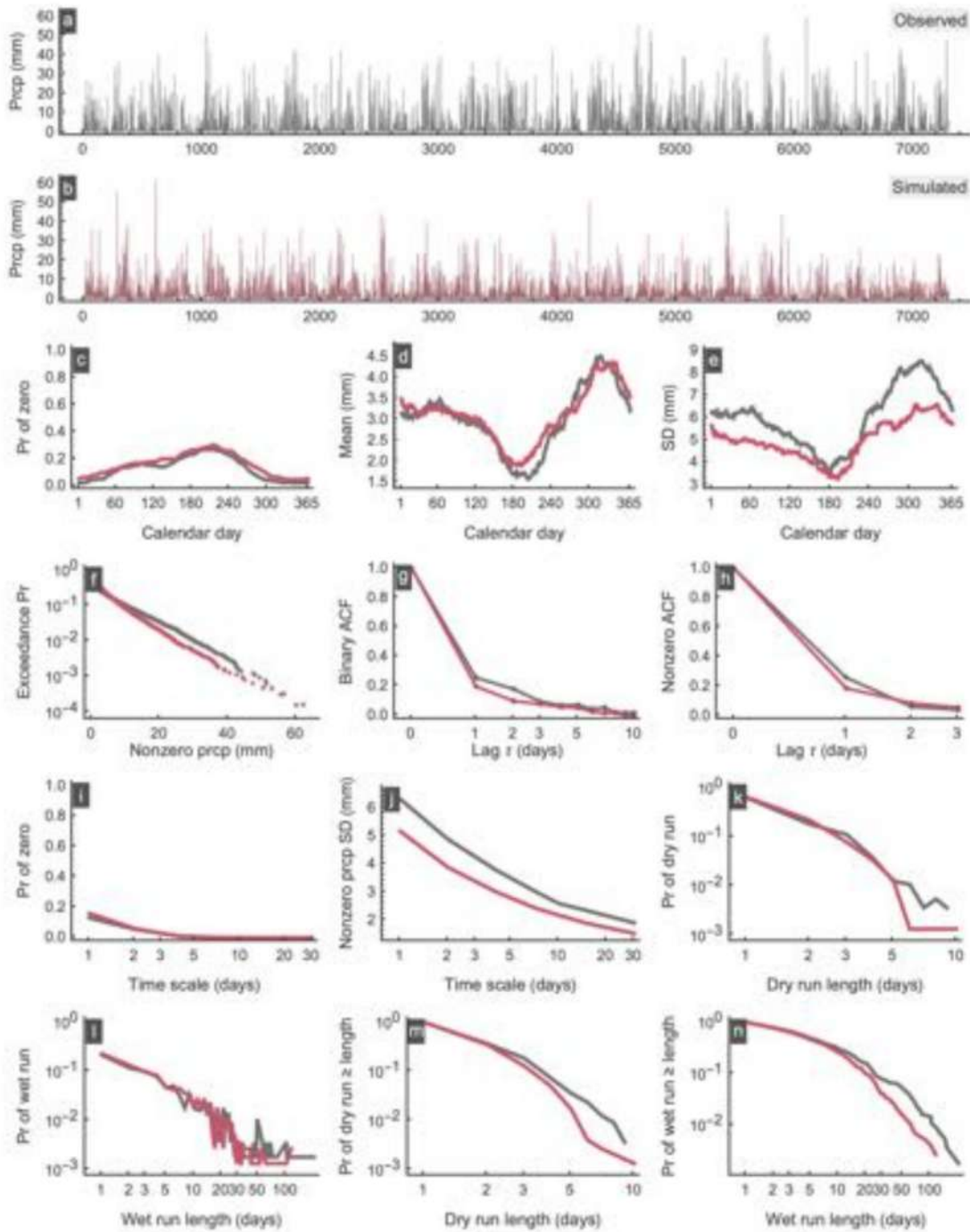
**Figure 3.** Comparison of observed and simulated station-level statistics: (a) probability of zero rainfall, (b) mean rainfall intensity, (c) L-variation, (d) L-skewness, (e) mean wet spell length, and (f) mean dry spell length.

**Catchment-scale simulation assessment** (Figure 4). This figure presents a detailed evaluation of the simulated and observed mean daily precipitation aggregated across all 70 stations, representing the catchment-scale behavior. The analysis follows the structure of the previous station-level multipanel plots and includes: (a–b) observed and simulated aggregated daily time series; (c) seasonal probability of zero precipitation; (d) seasonal mean rainfall intensity; (e) seasonal standard deviation of rainfall intensity; (f) exceedance probability of nonzero rainfall; (g) binary

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autocorrelation structure; (h) nonzero rainfall autocorrelation structure; (i) scaling of the probability of zero rainfall with time scale; (j) scaling of standard deviation; (k) probability mass function of dry spell lengths; (l) probability mass function of wet spell lengths; (m) complementary cumulative distribution of dry spell lengths; and (n) complementary cumulative distribution of wet spell lengths.

This assessment demonstrates that the simulation effectively reproduces key statistical properties of rainfall at the catchment scale, including variability, persistence, and extremes. The agreement between observed and simulated series is excellent across all evaluated metrics. The only noticeable deviation is a slight underestimation of the standard deviation of the aggregated catchment rainfall (panel e), which may reflect the smoothing effect introduced during spatial aggregation. Nonetheless, the overall alignment confirms the robustness and reliability of the simulation framework at both the station and catchment levels.



**Figure 4.** Assessment of observed and simulated catchment-scale mean daily precipitation aggregated from all 70 stations. Panels include (a–b) daily time series, (c) seasonal probability of zero, (d) seasonal mean, (e) seasonal standard deviation, (f) exceedance probability, (g–h) autocorrelation structures, (i–j) scaling metrics, and (k–n) dry/wet spell statistics. See text for details.

## Appendix A: Additional Figures

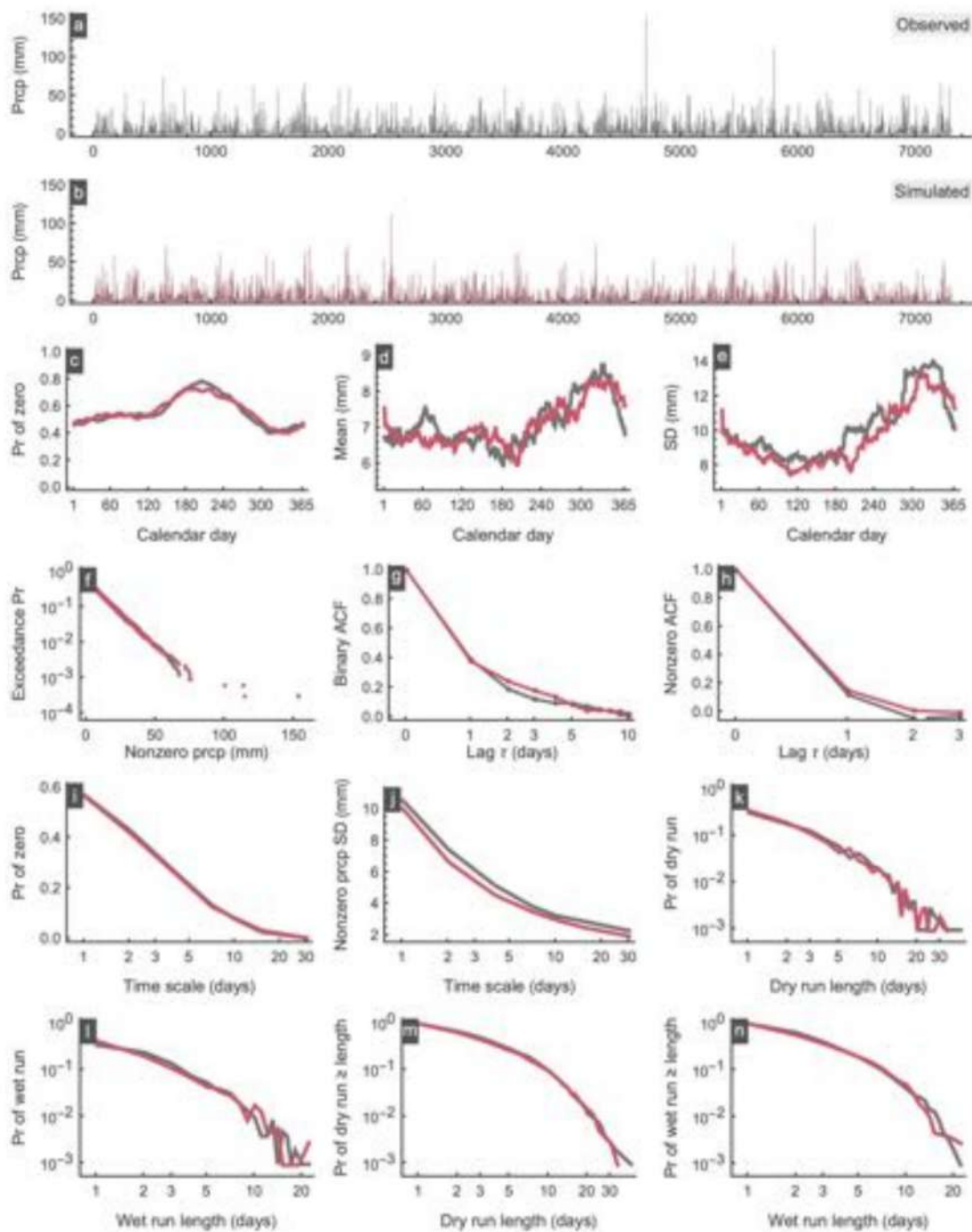


Figure A1. Simulation results for station PluvioMetro 00.

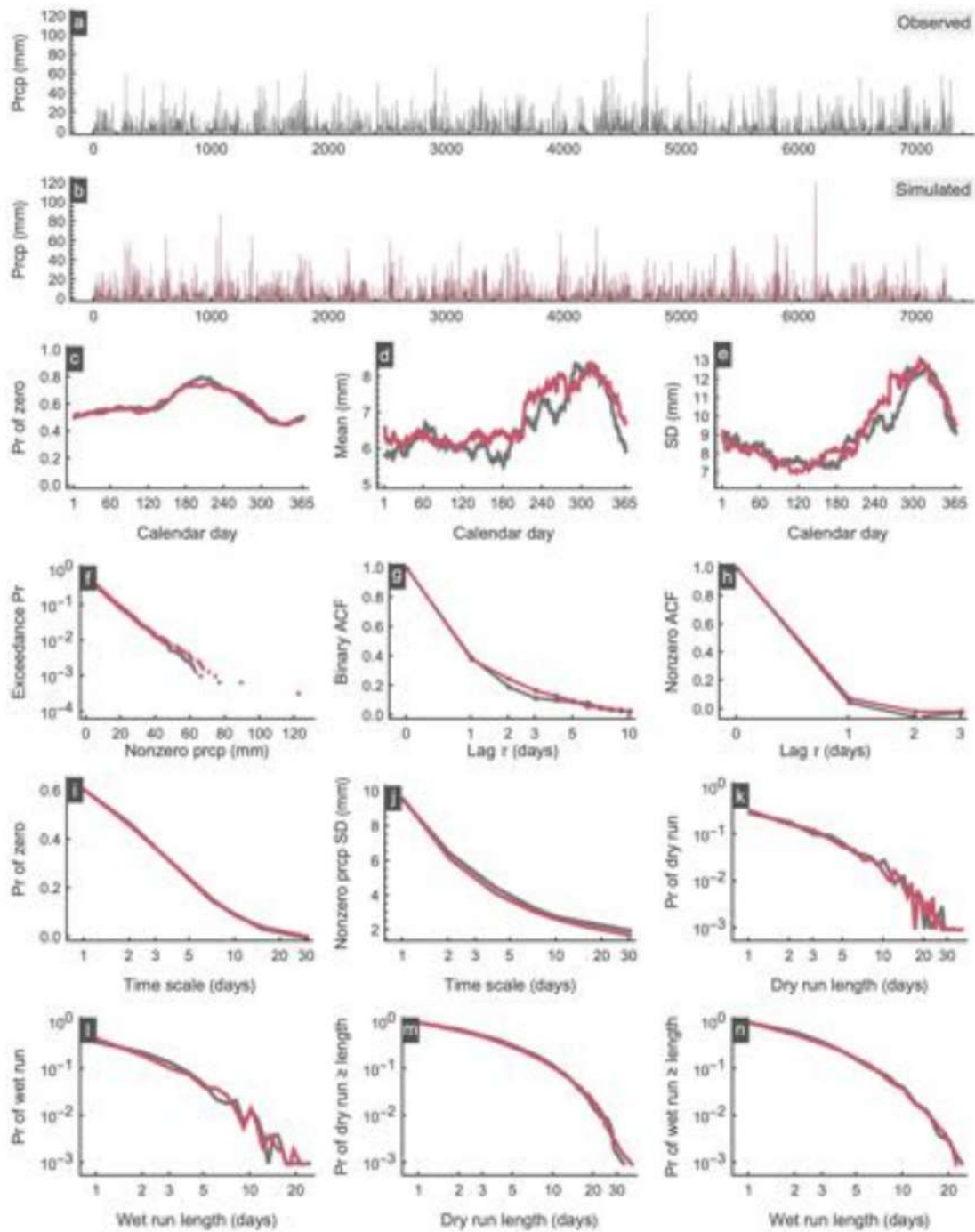


Figure A2. Simulation results for station PluvioMetro 01.

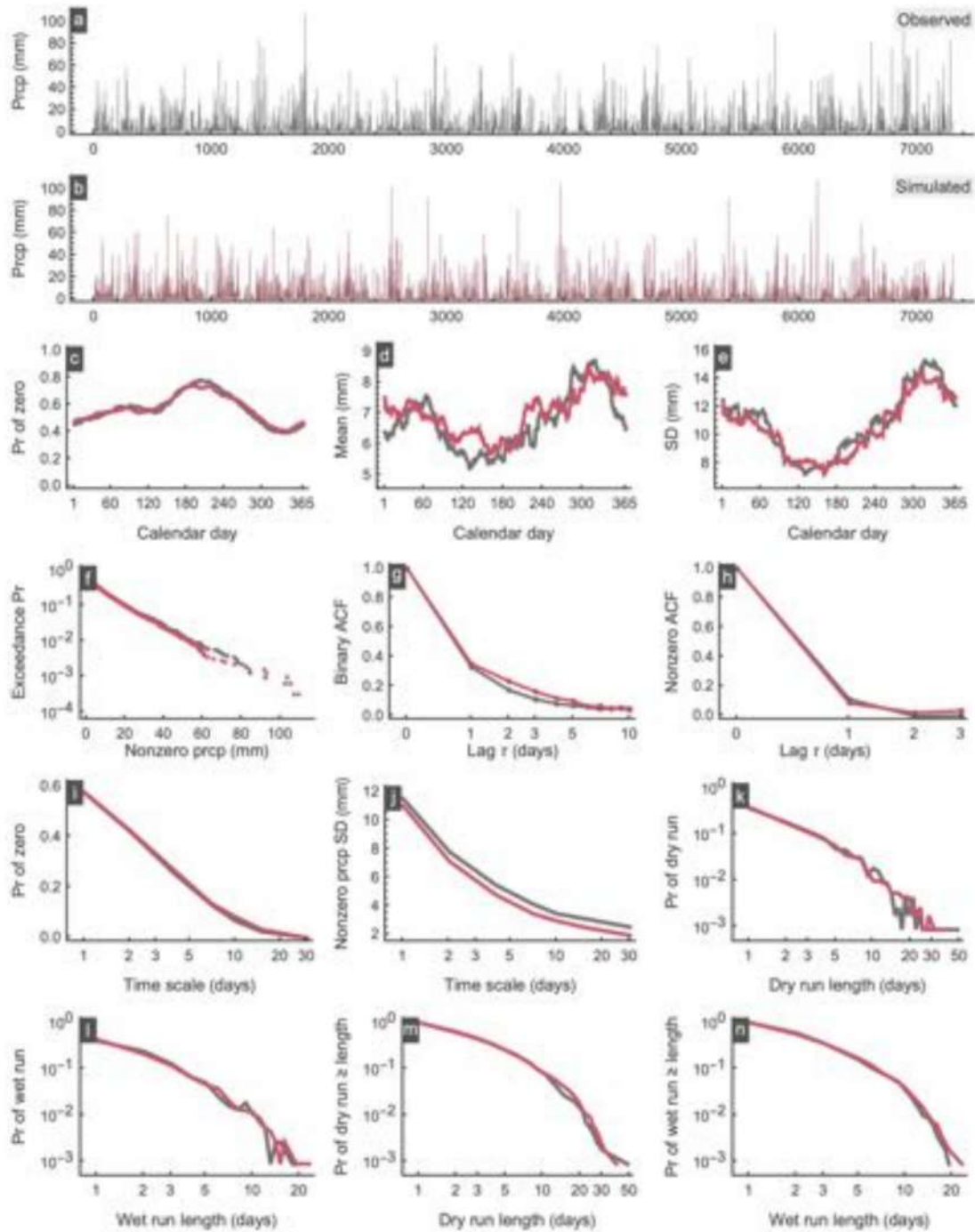


Figure A3. Simulation results for station PluvioMetro 02.

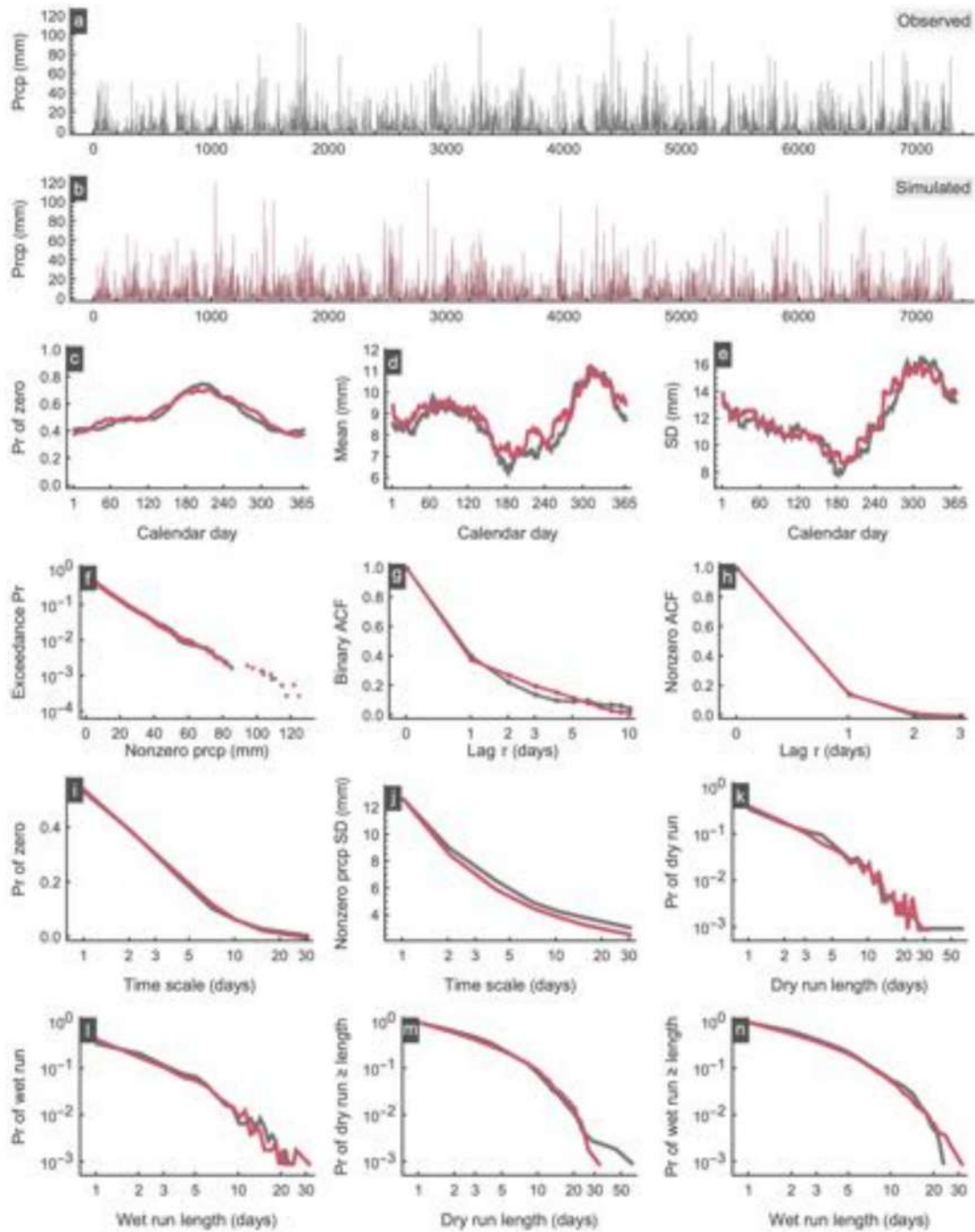


Figure A4. Simulation results for station PluvioMetro 03.

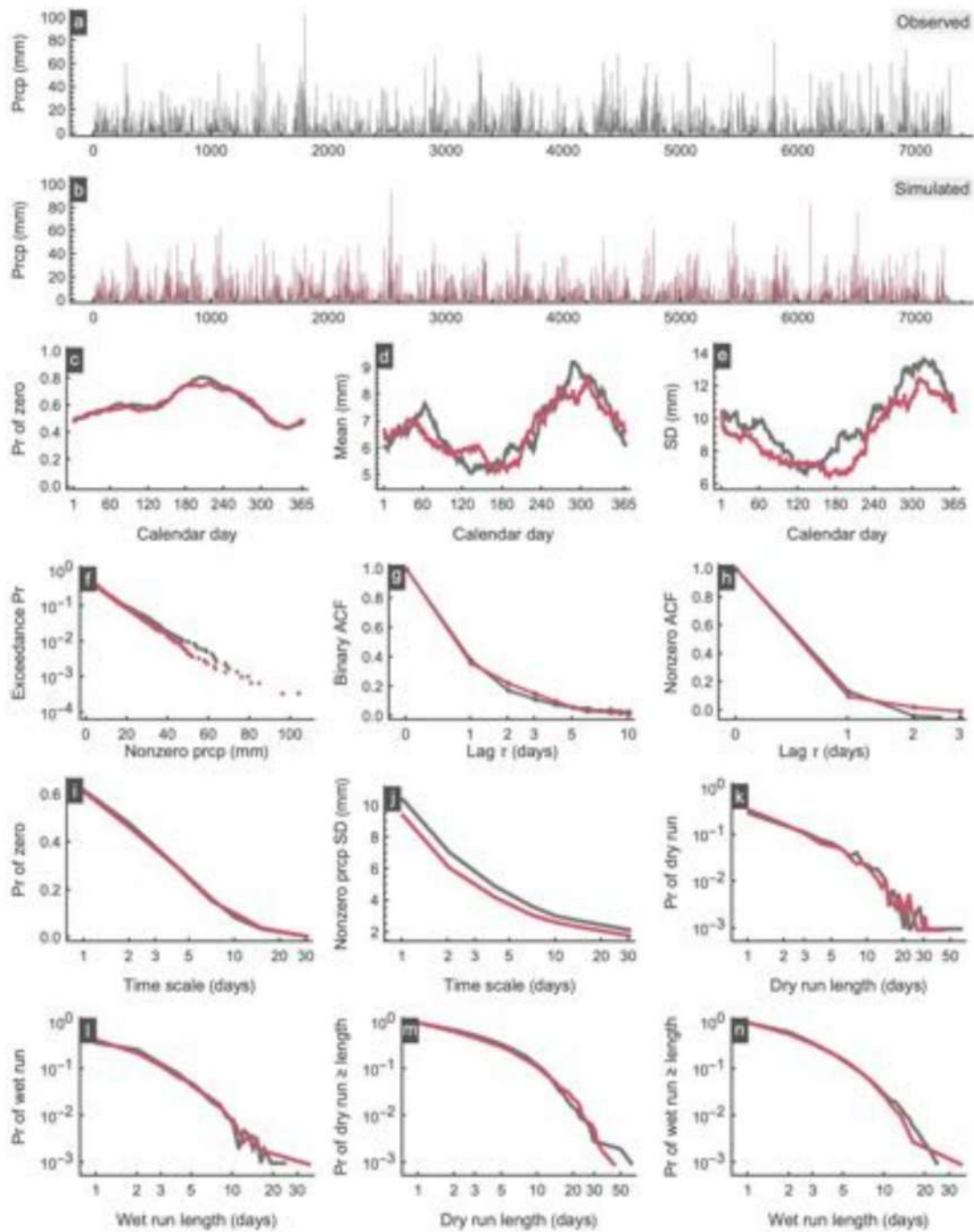


Figure A5. Simulation results for station PluvioMetro 04.

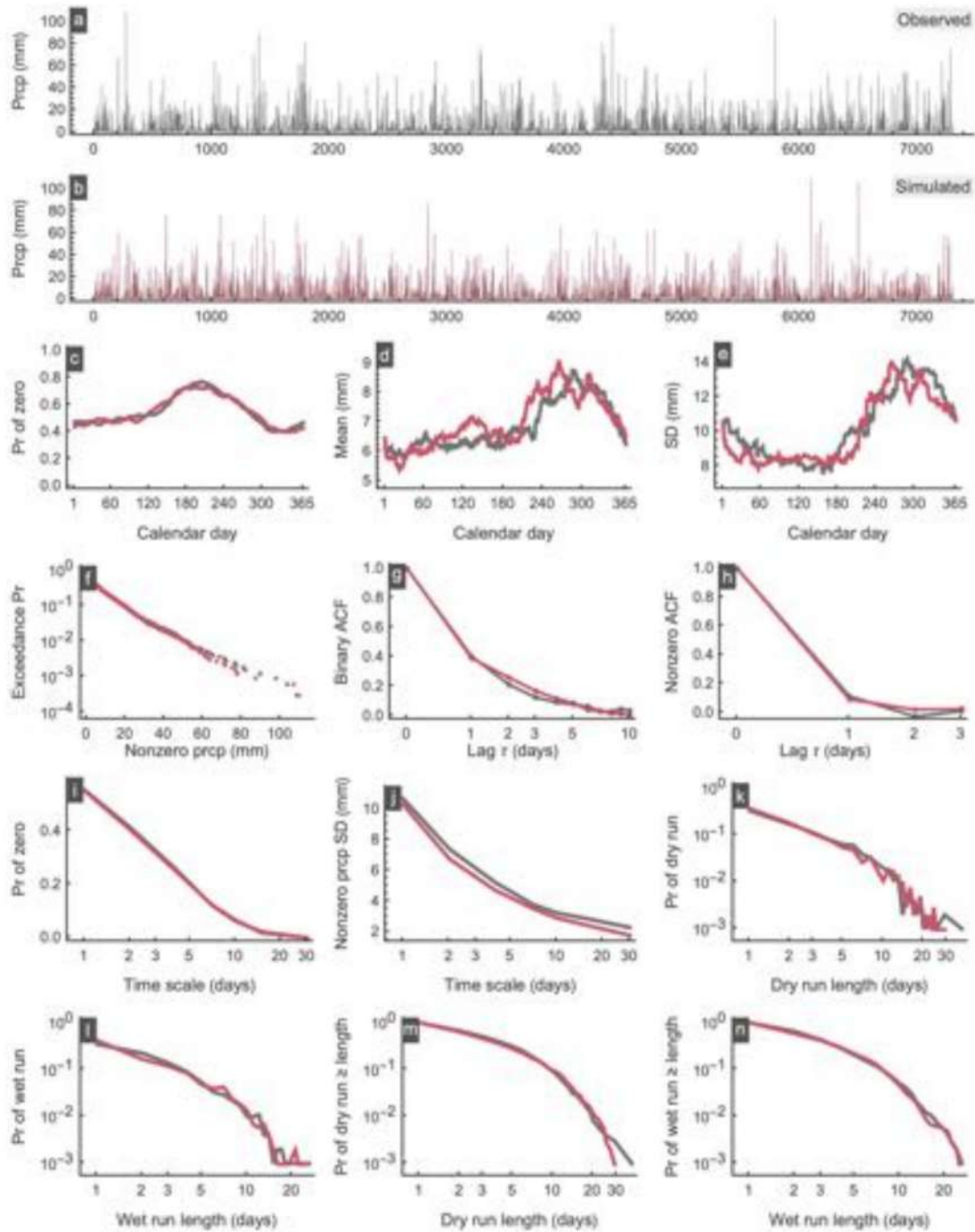


Figure A6. Simulation results for station PluvioMetro 05.

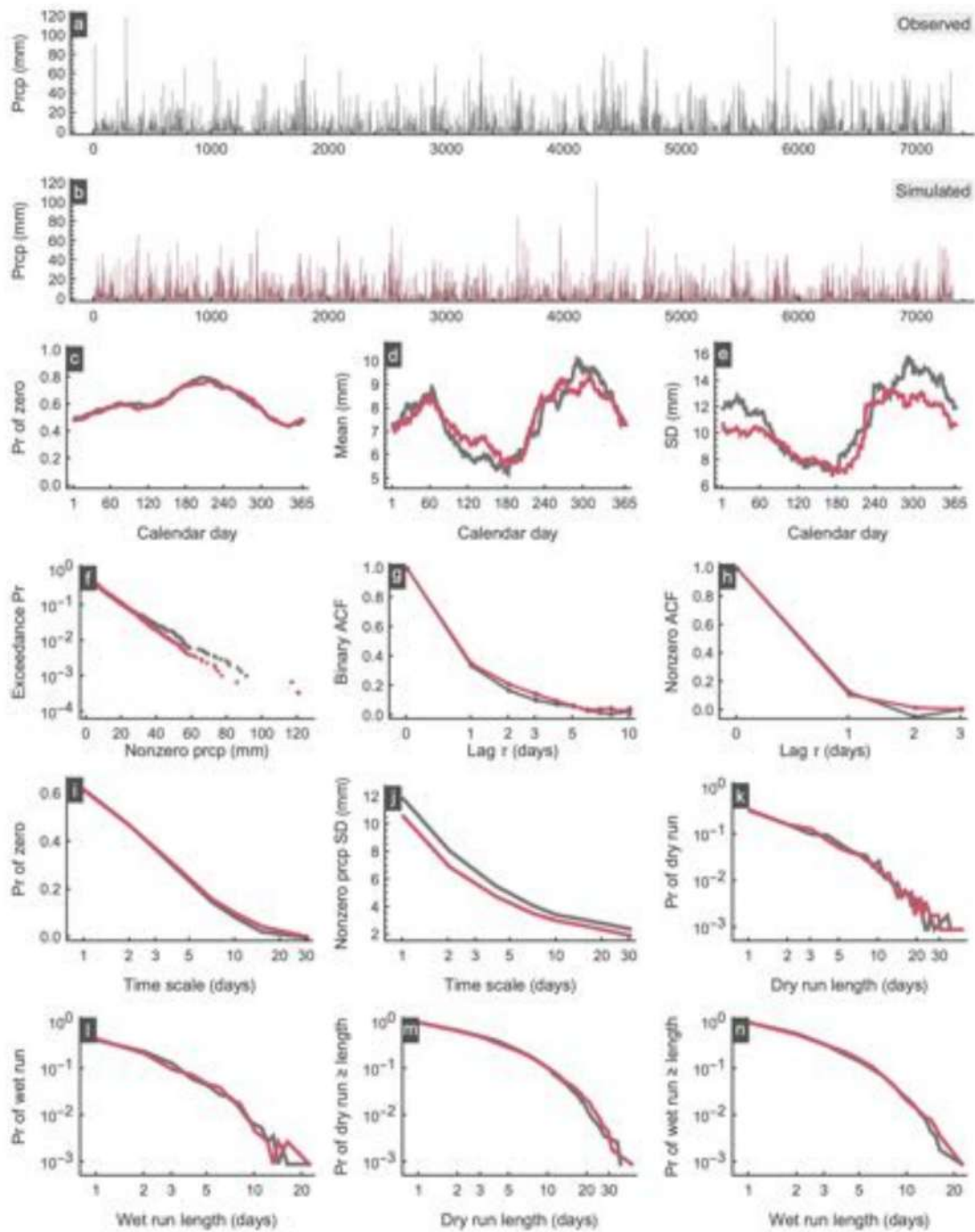


Figure A7. Simulation results for station PluvioMetro 06.

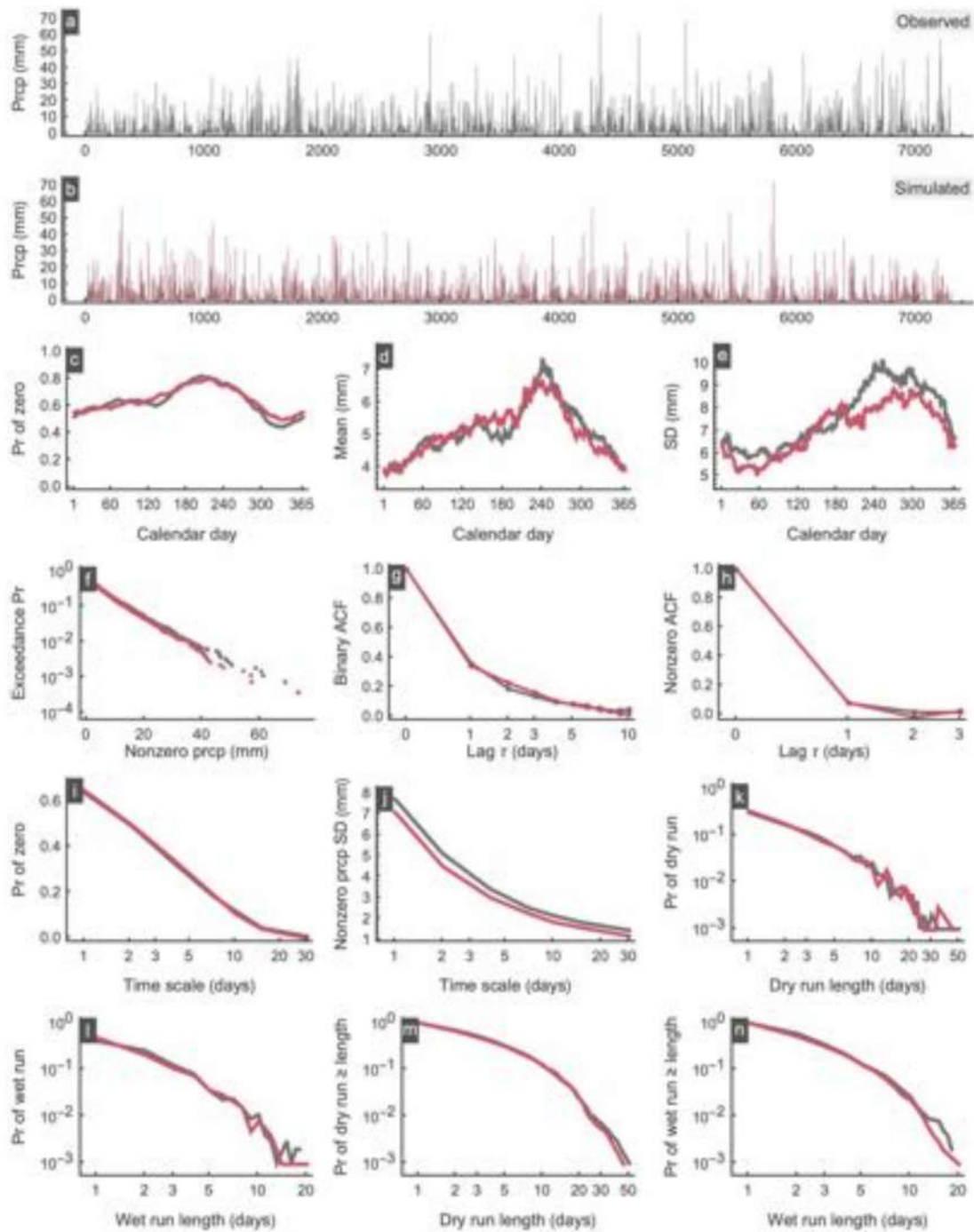


Figure A8. Simulation results for station PluvioMetro 07.

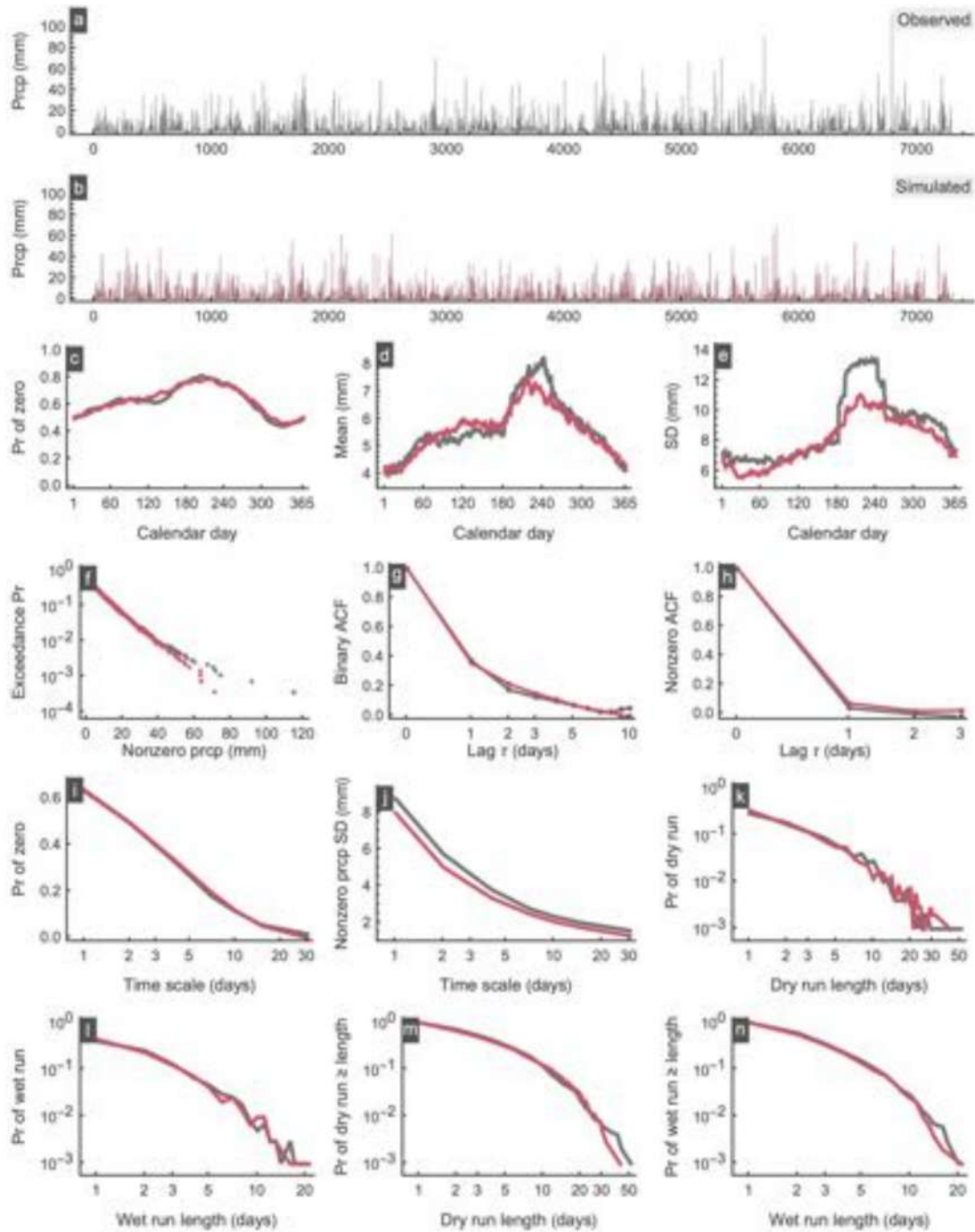


Figure A9. Simulation results for station PluvioMetro 08.

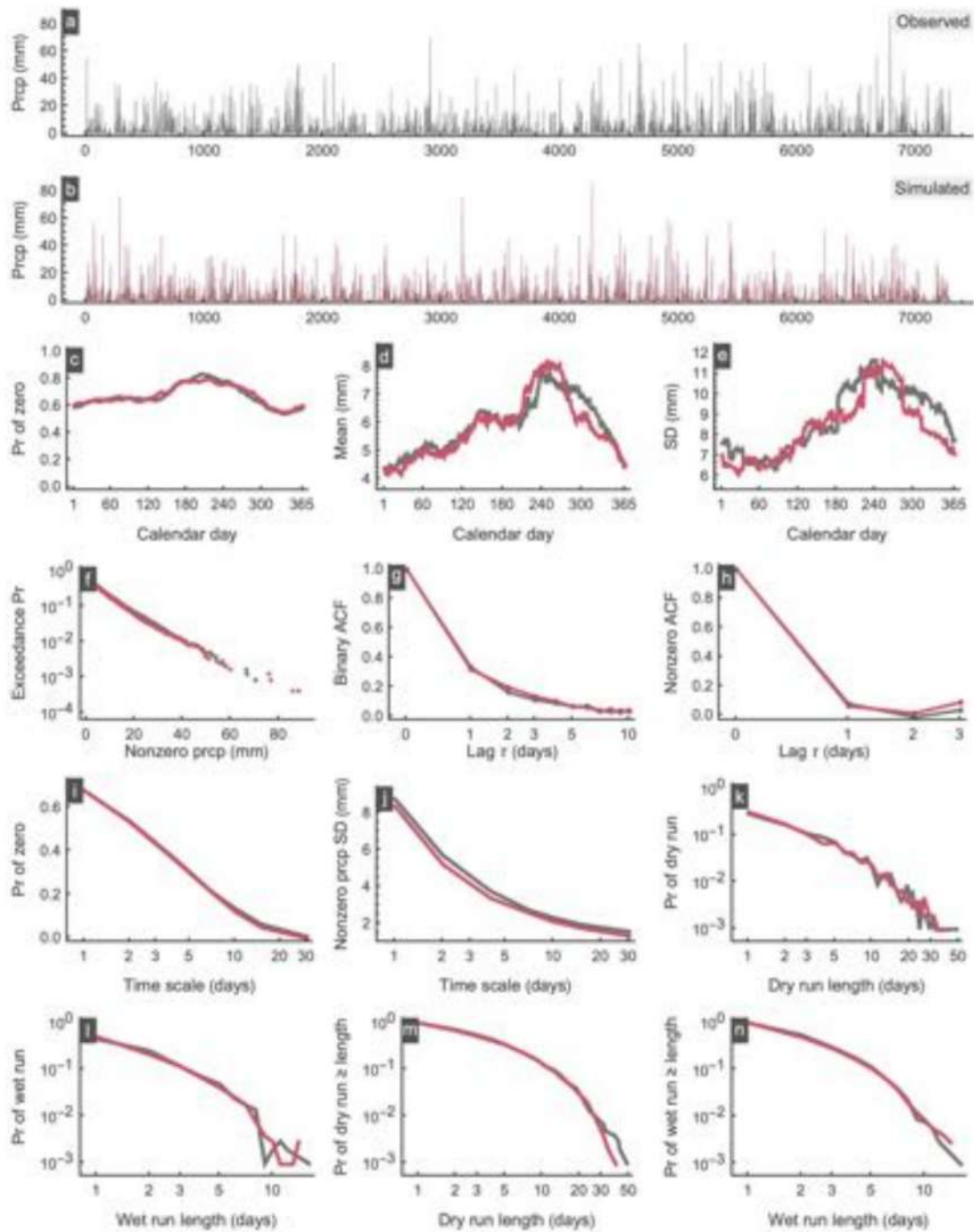


Figure A10. Simulation results for station PluvioMetro 11.

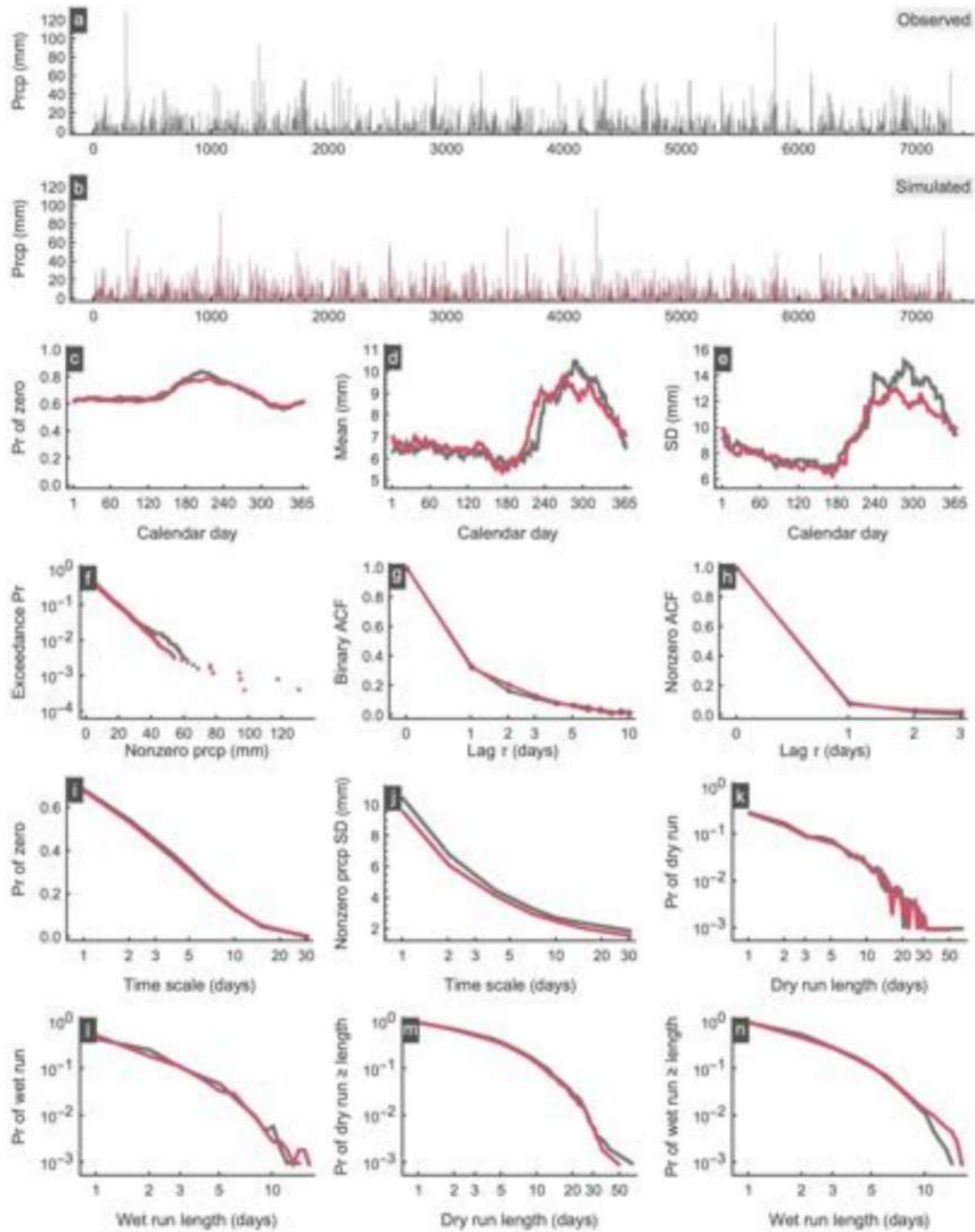


Figure A11. Simulation results for station PluvioMetro 13.

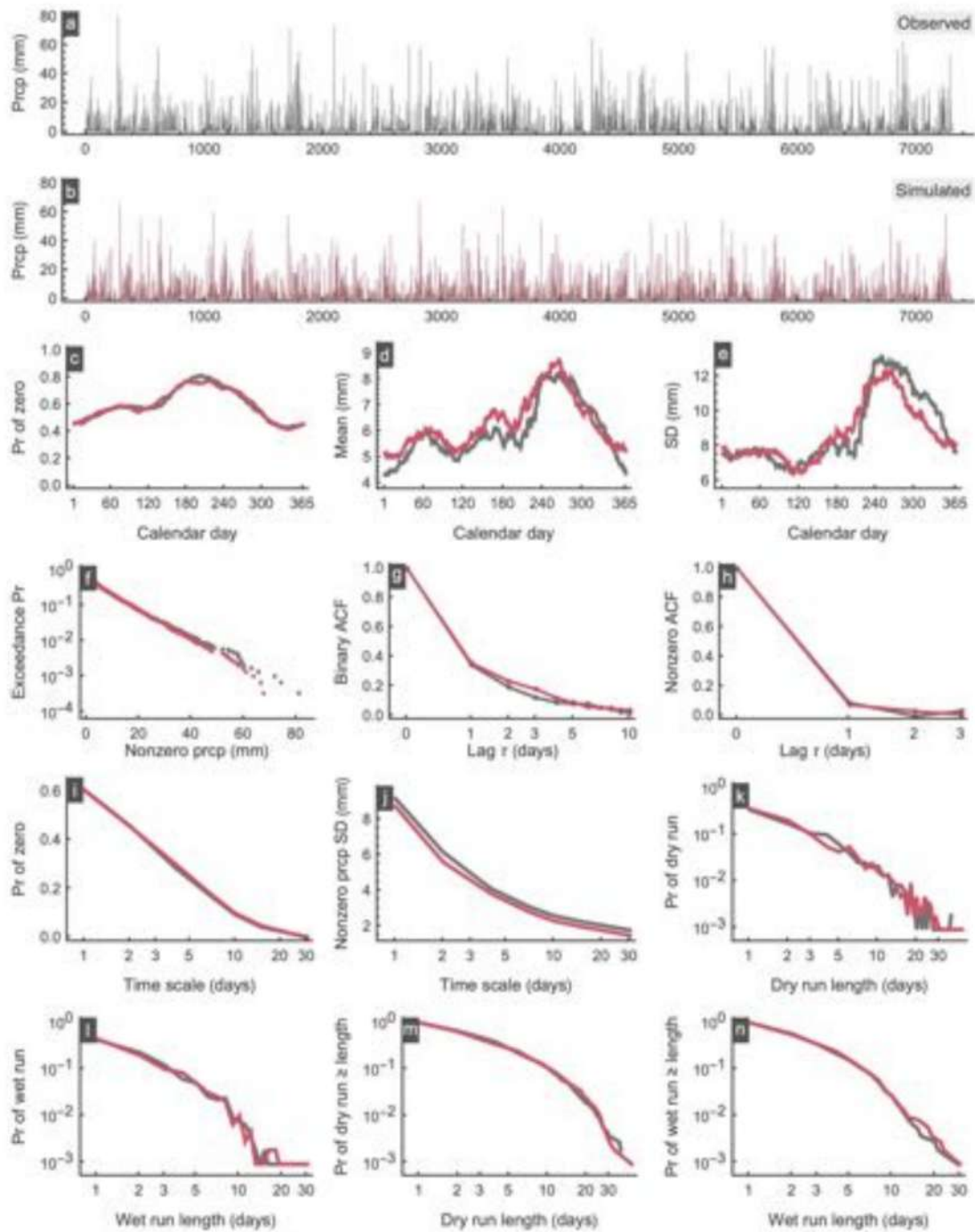


Figure A12. Simulation results for station PluvioMetro 14.

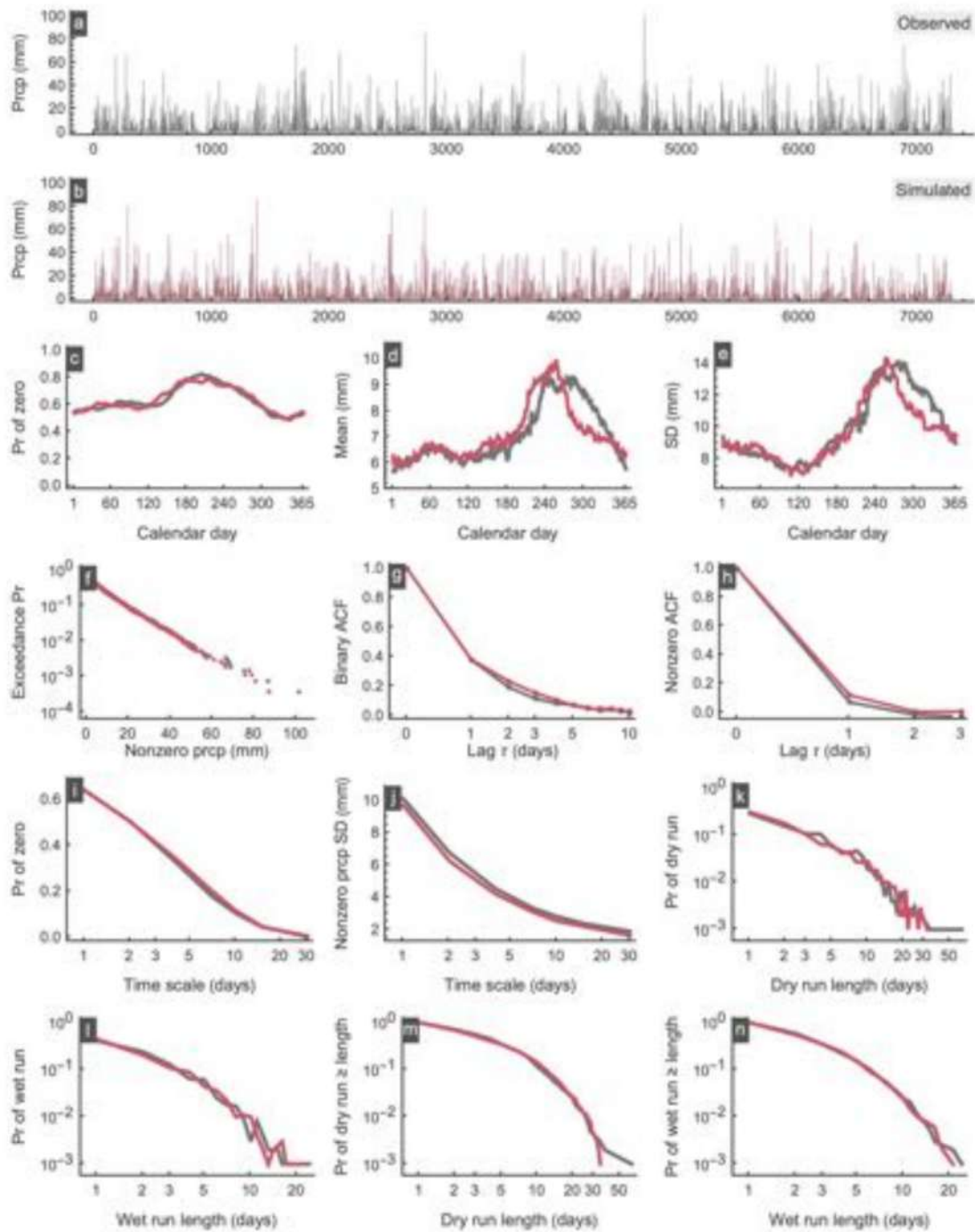


Figure A13. Simulation results for station PluvioMetro 15.

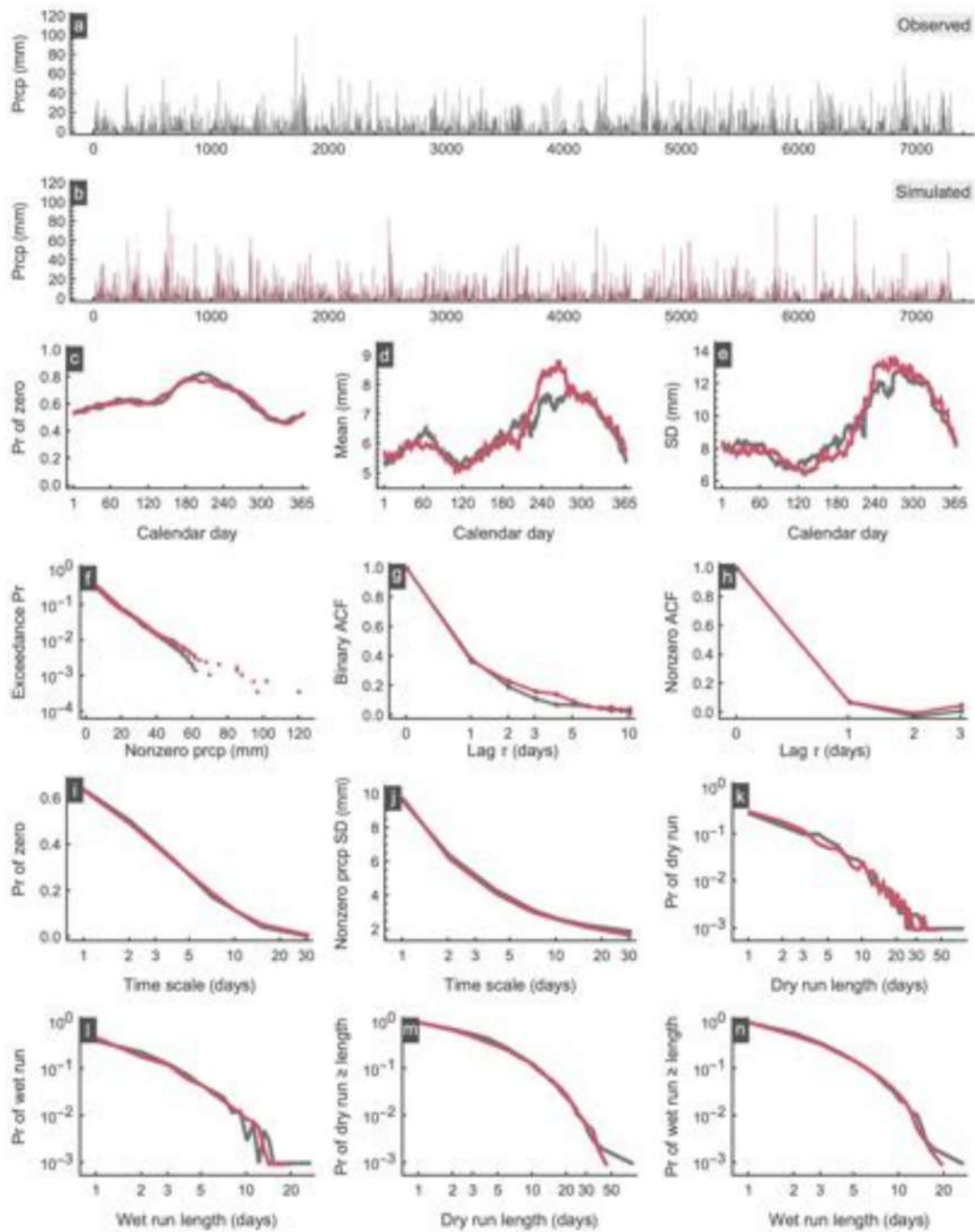


Figure A14. Simulation results for station PluvioMetro 16.

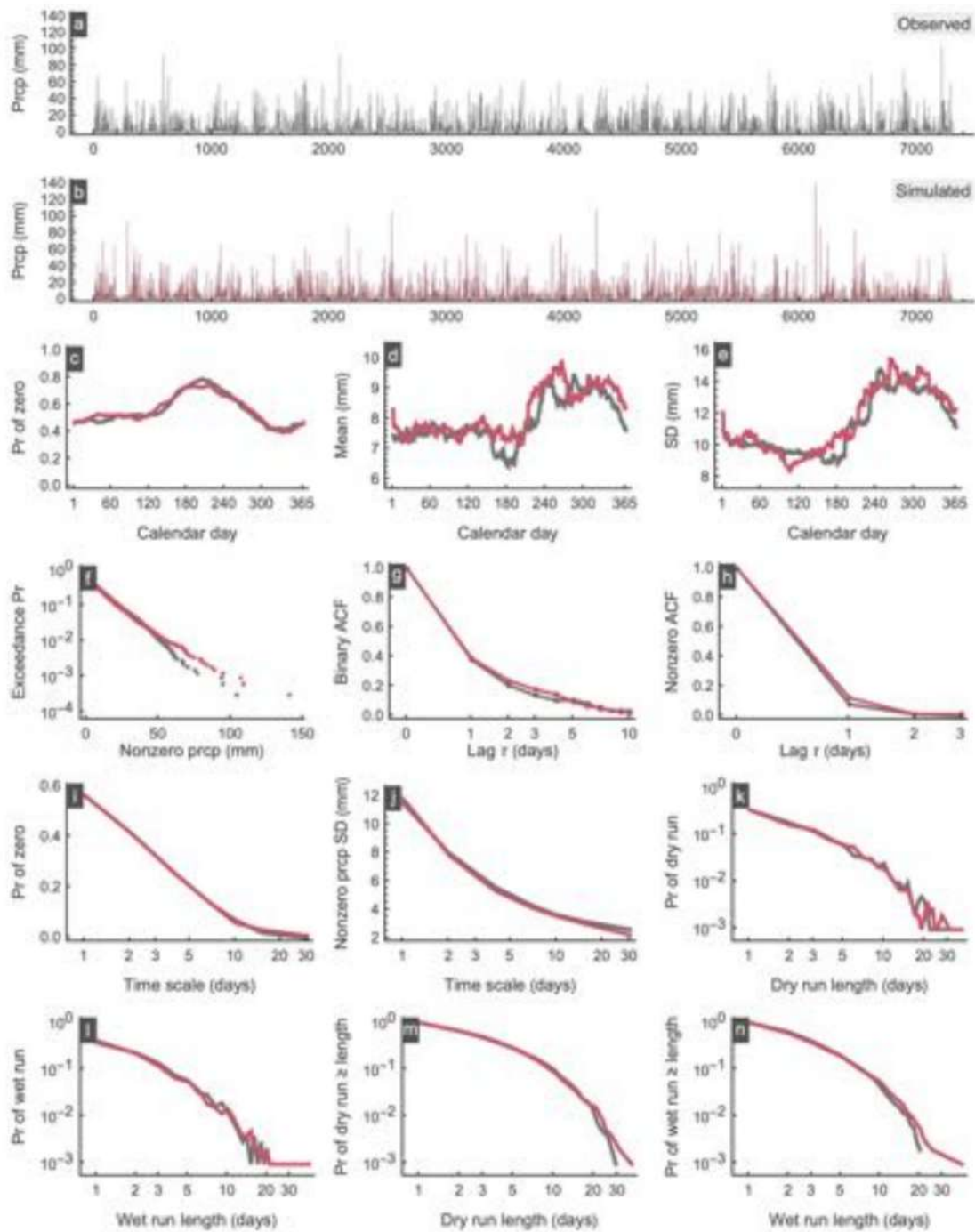


Figure A15. Simulation results for station PluvioMetro 17.

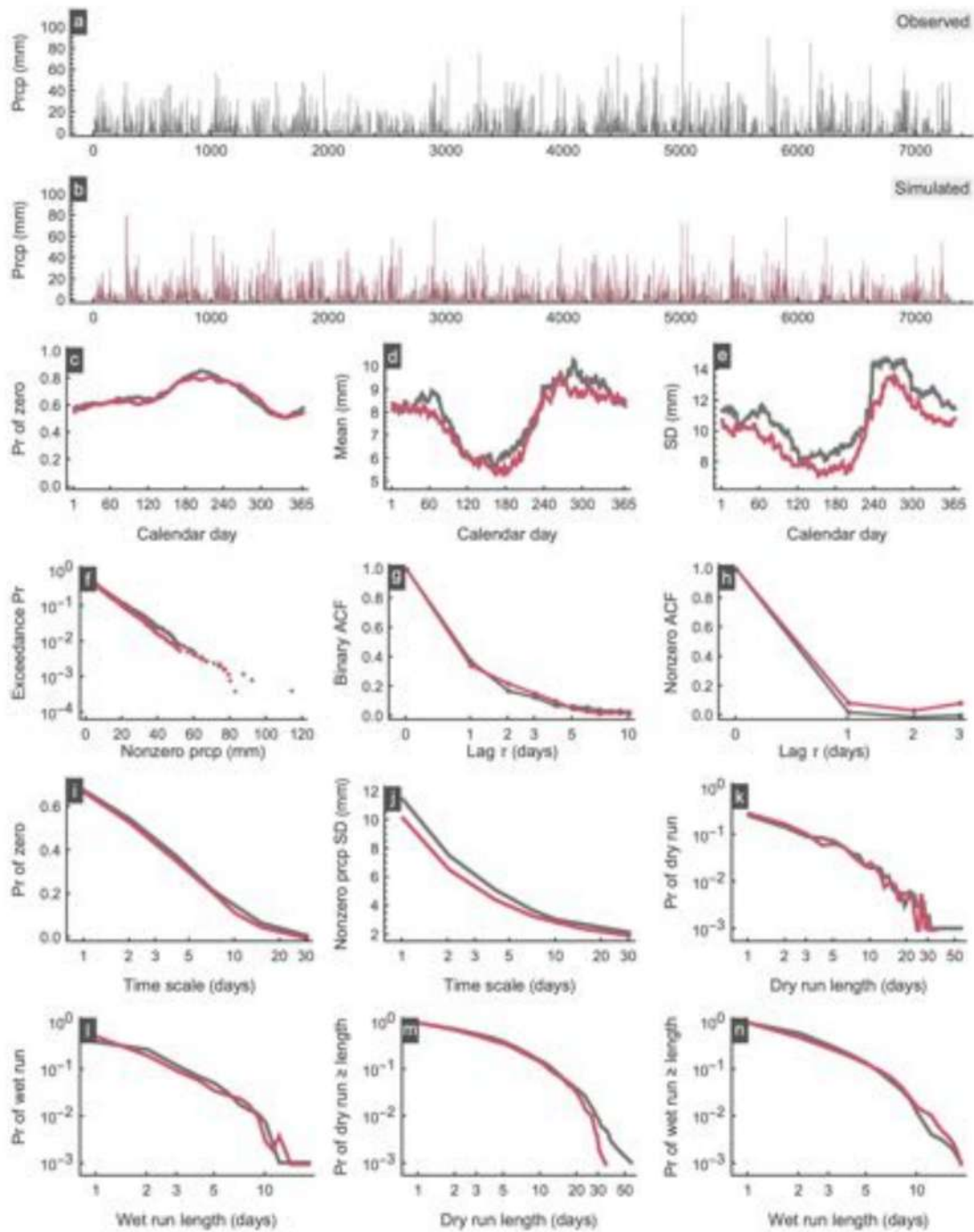


Figure A16. Simulation results for station PluvioMetro 18.

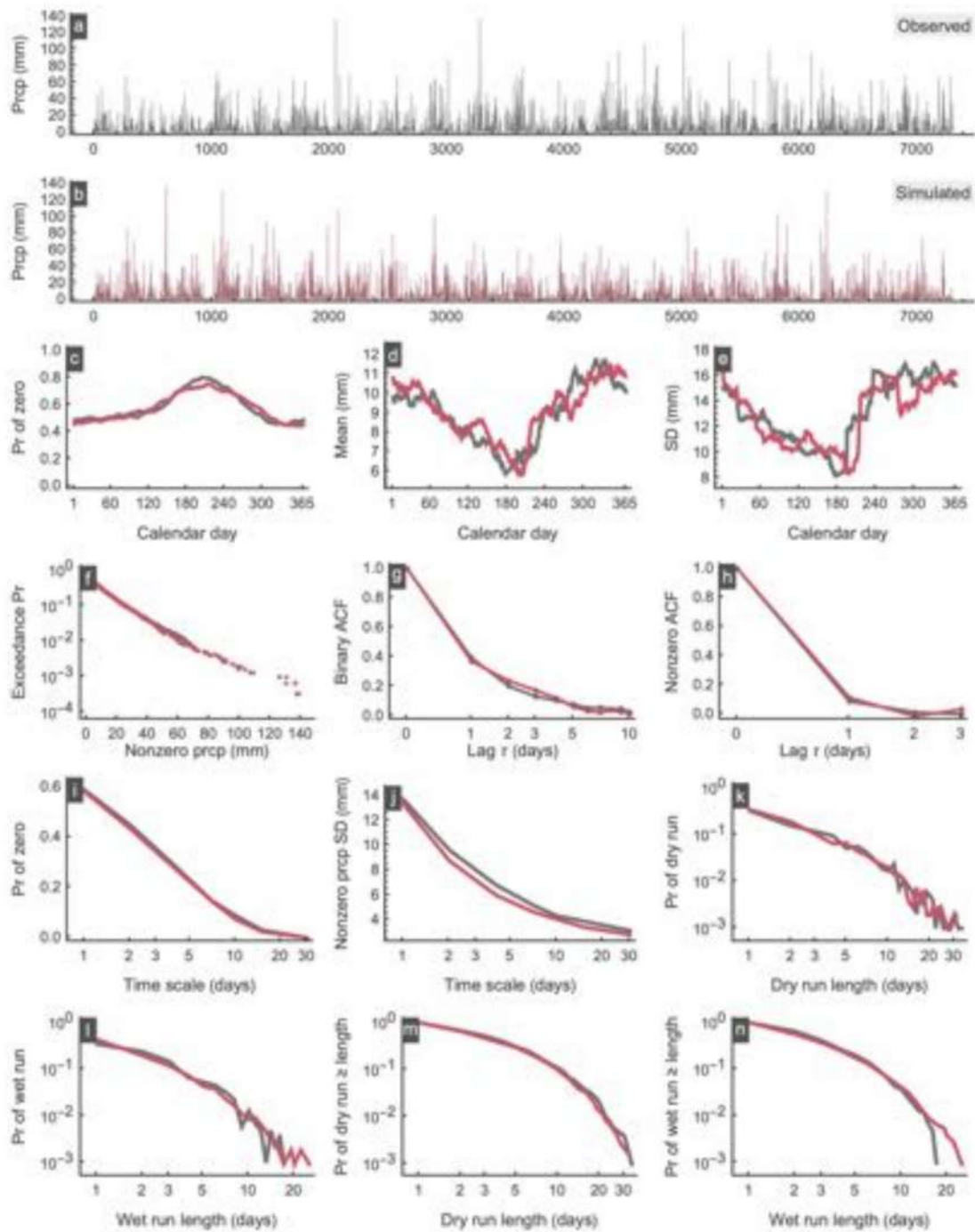


Figure A17. Simulation results for station PluvioMetro 19.

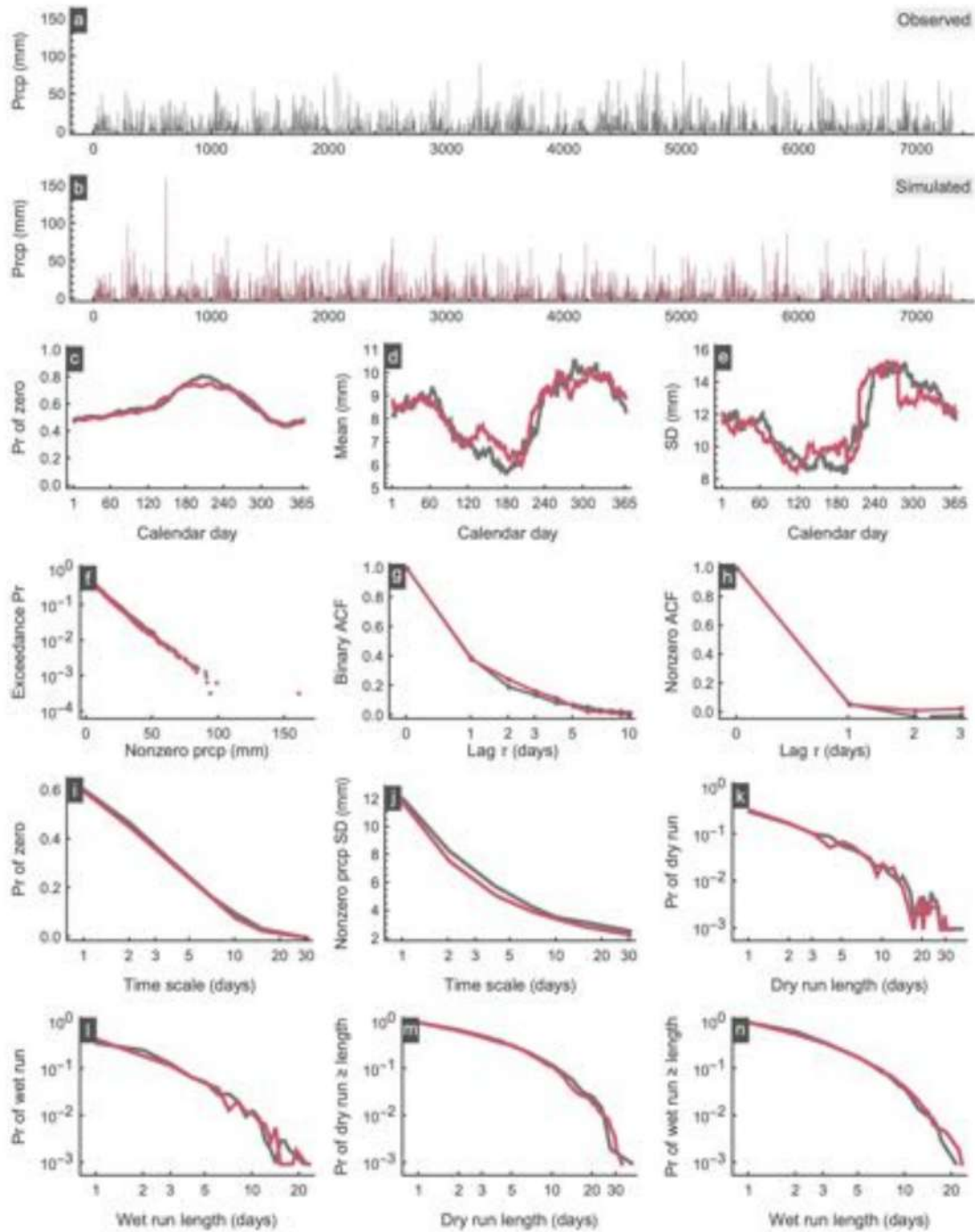


Figure A18. Simulation results for station PluvioMetro 21.

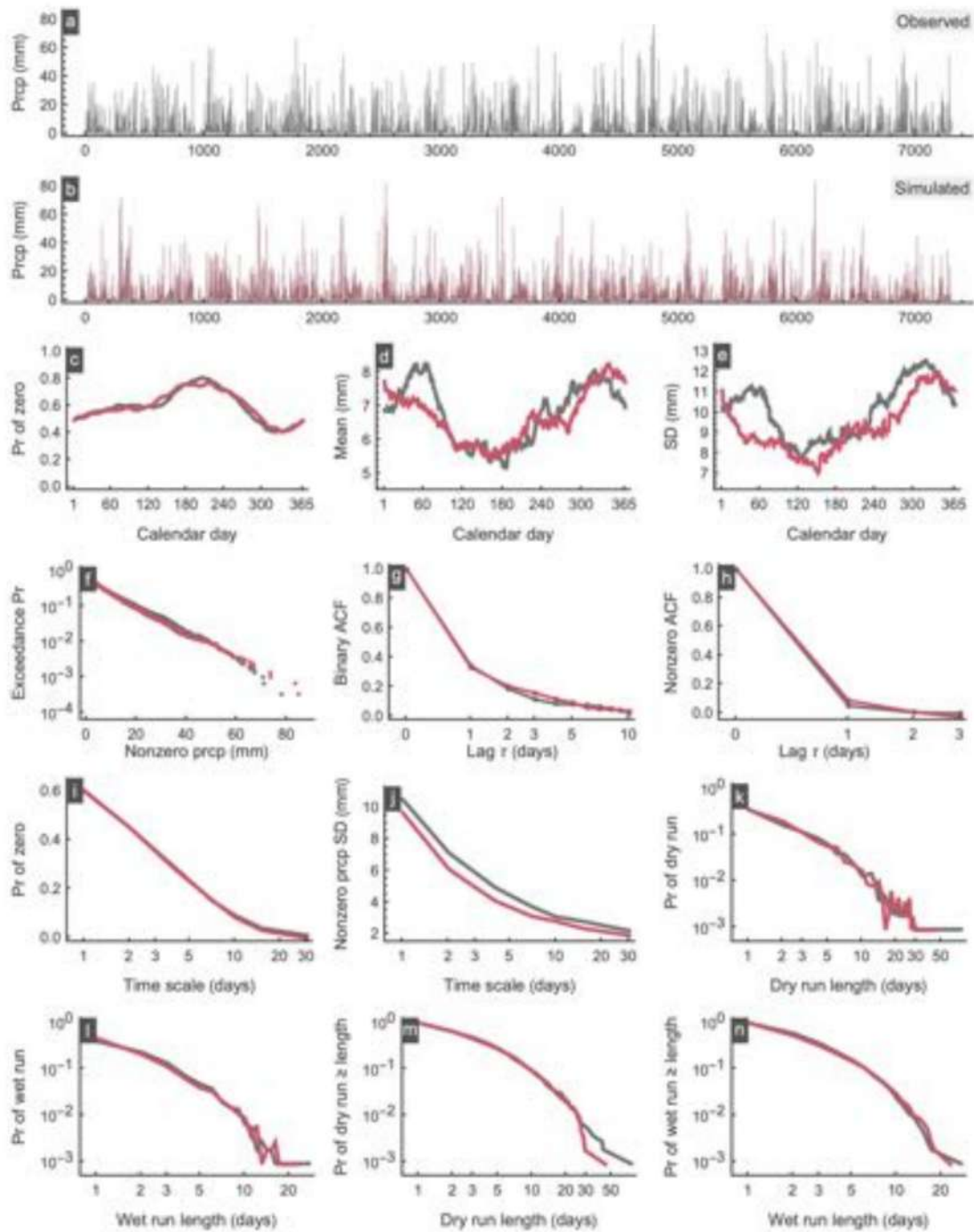


Figure A19. Simulation results for station PluvioMetro 22.

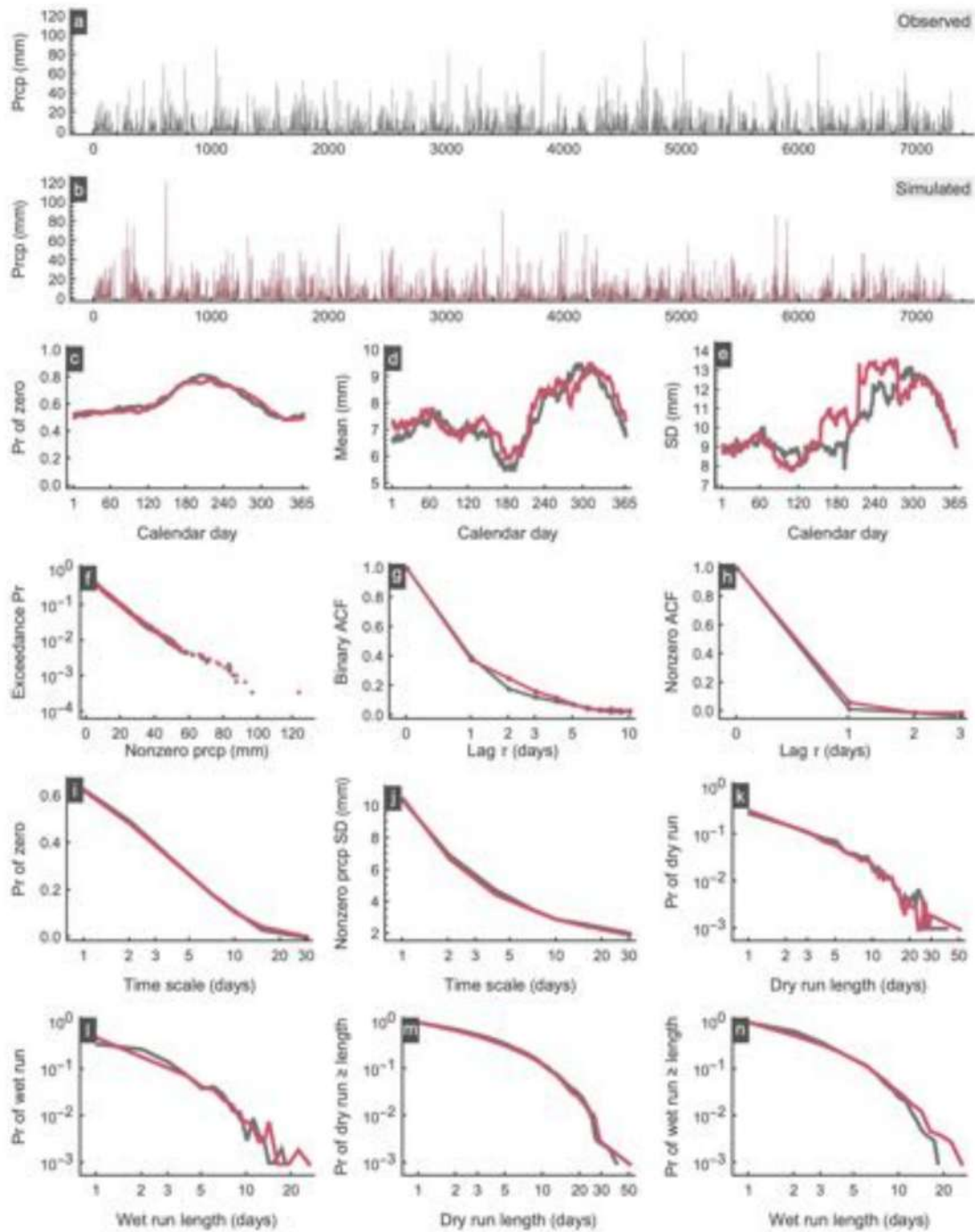


Figure A20. Simulation results for station PluvioMetro 24.

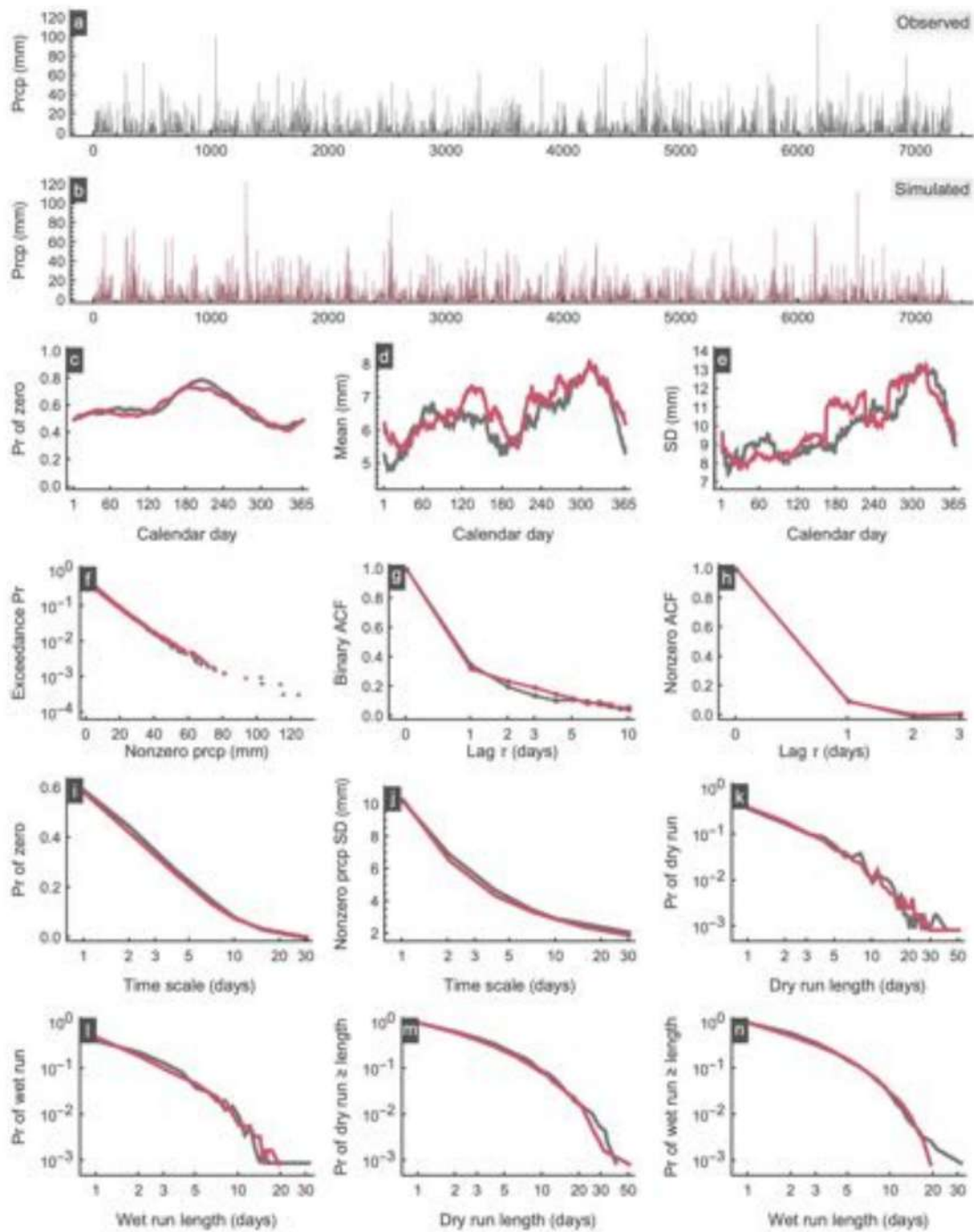


Figure A21. Simulation results for station PluvioMetro 26.

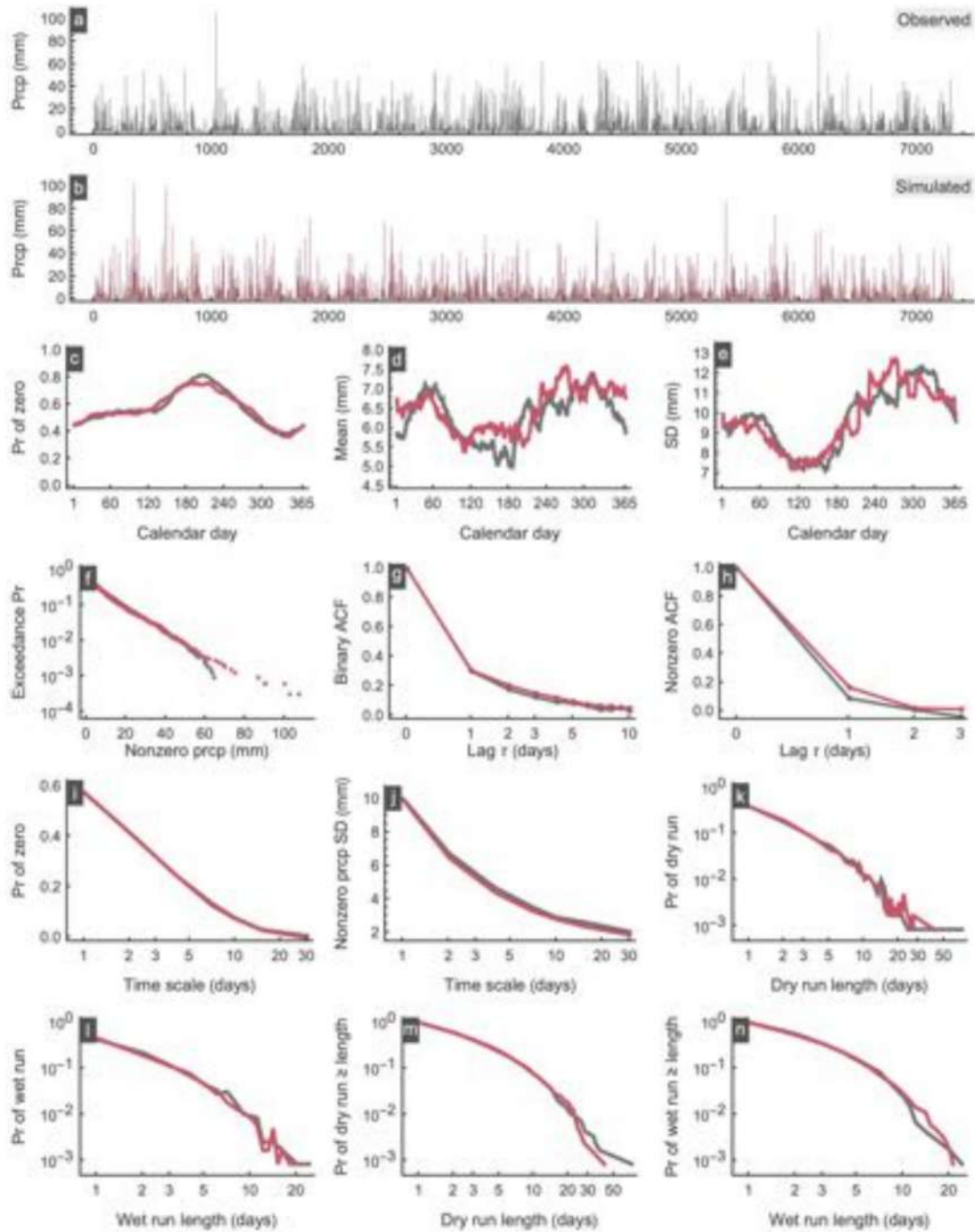


Figure A22. Simulation results for station PluvioMetro 27.

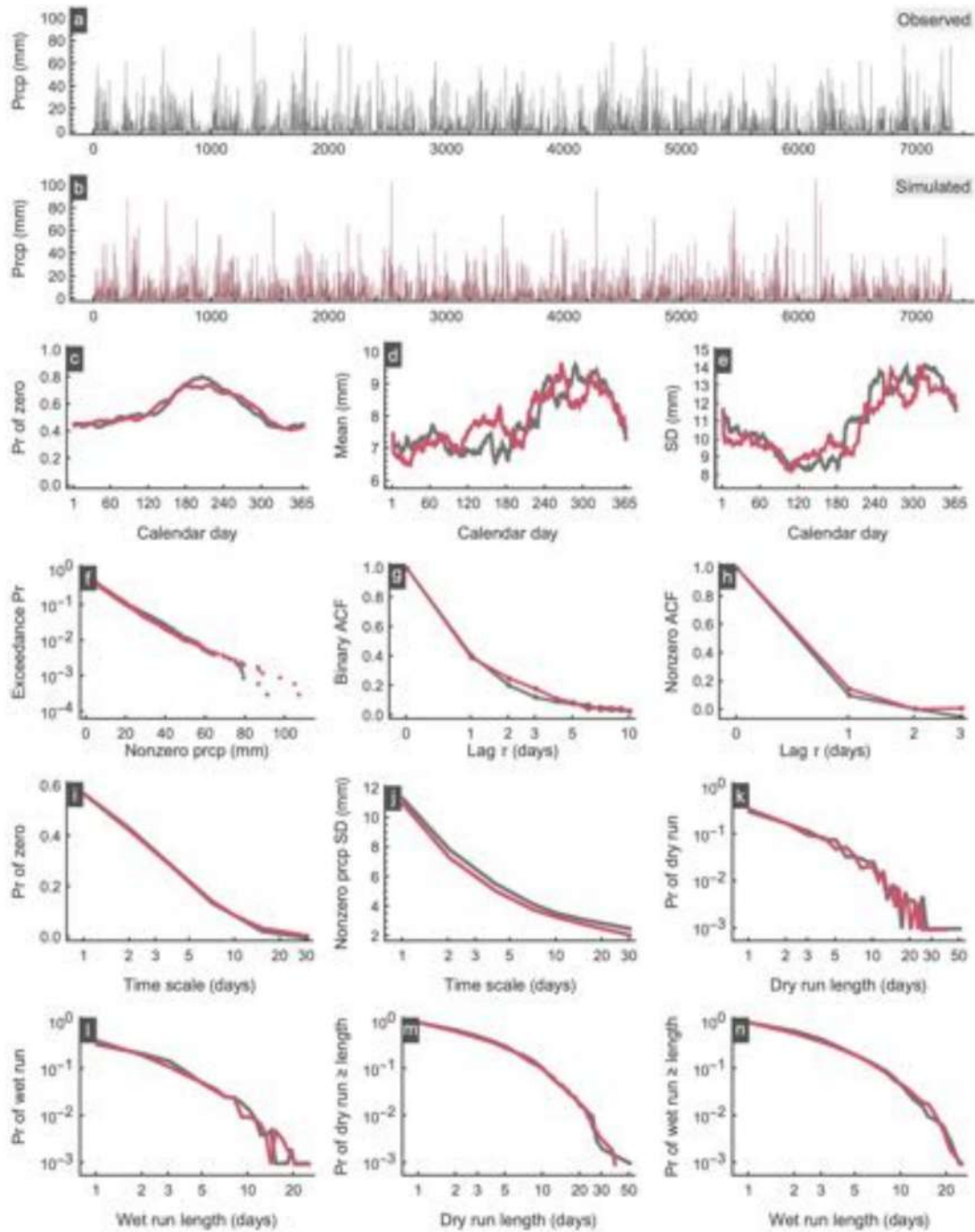


Figure A23. Simulation results for station PluvioMetro 28.

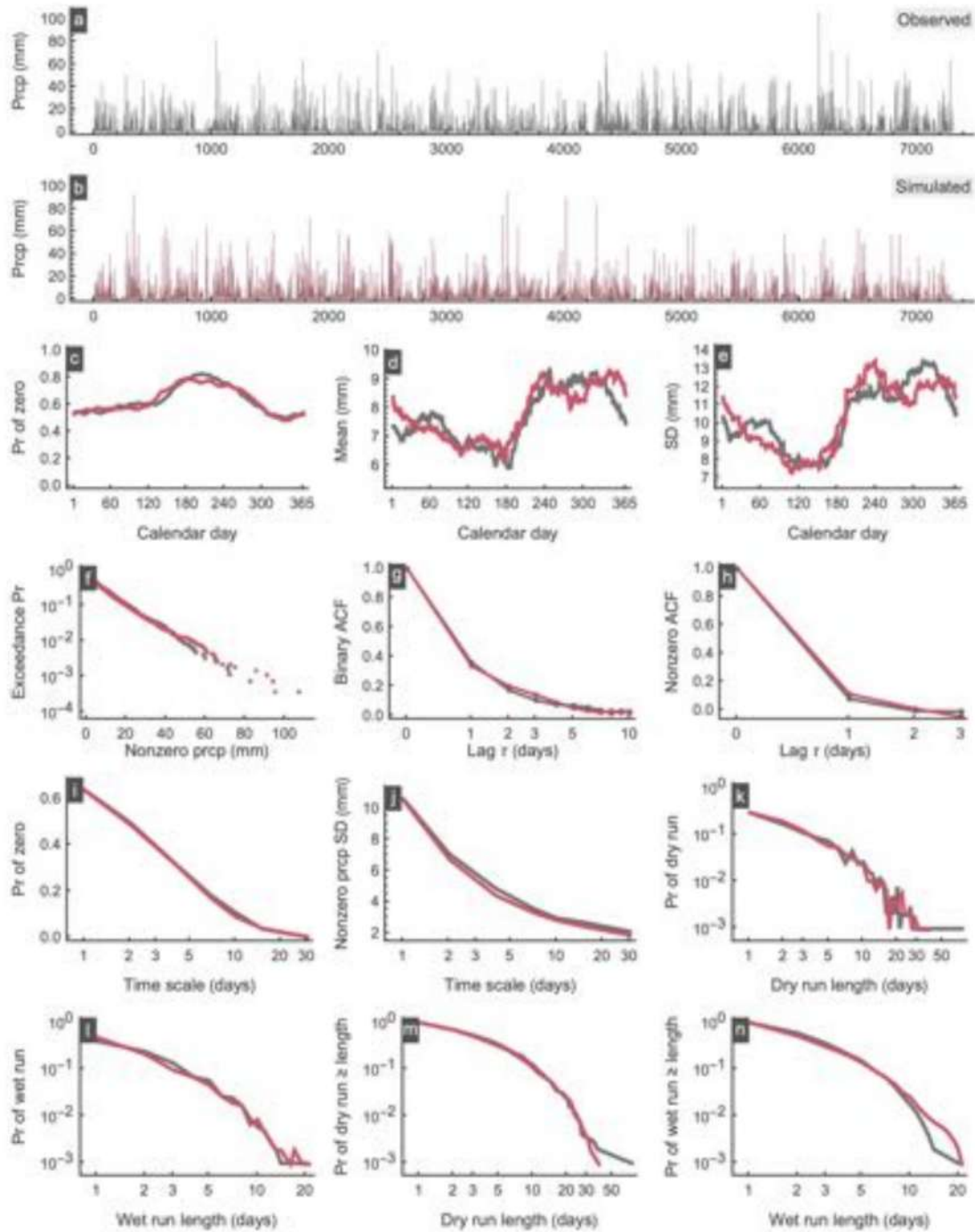


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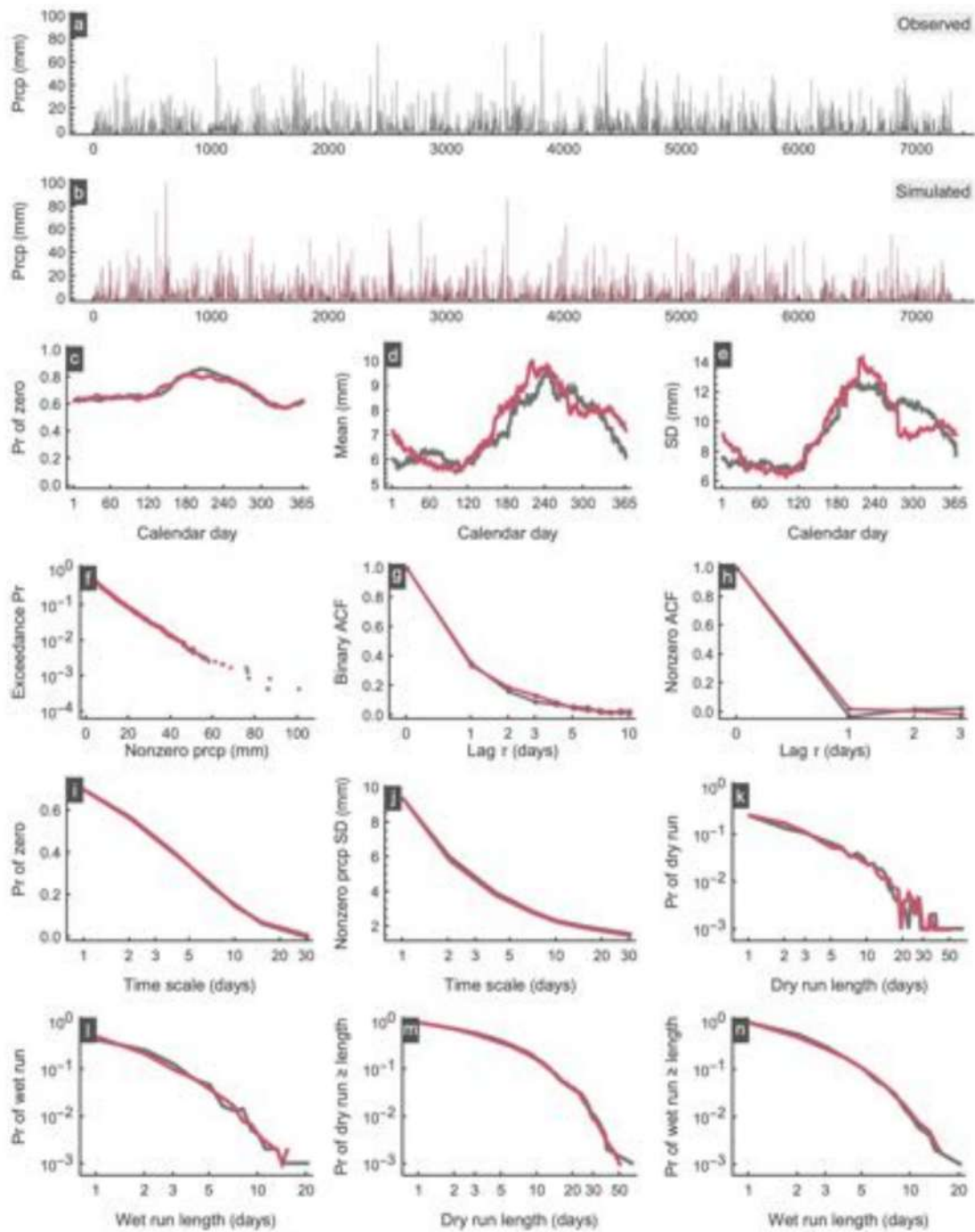


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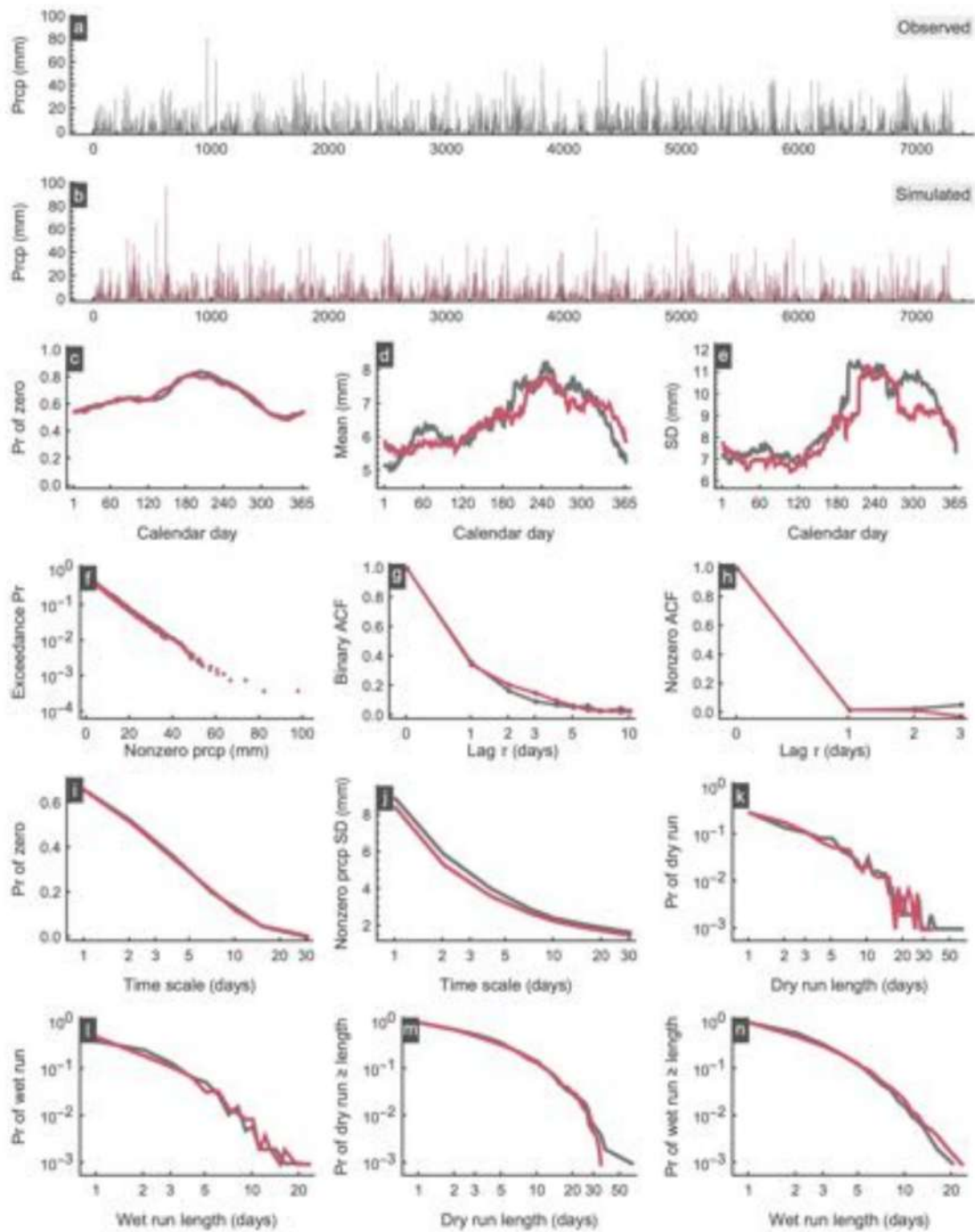


Figure A26. Simulation results for station PluvioMetro 31.

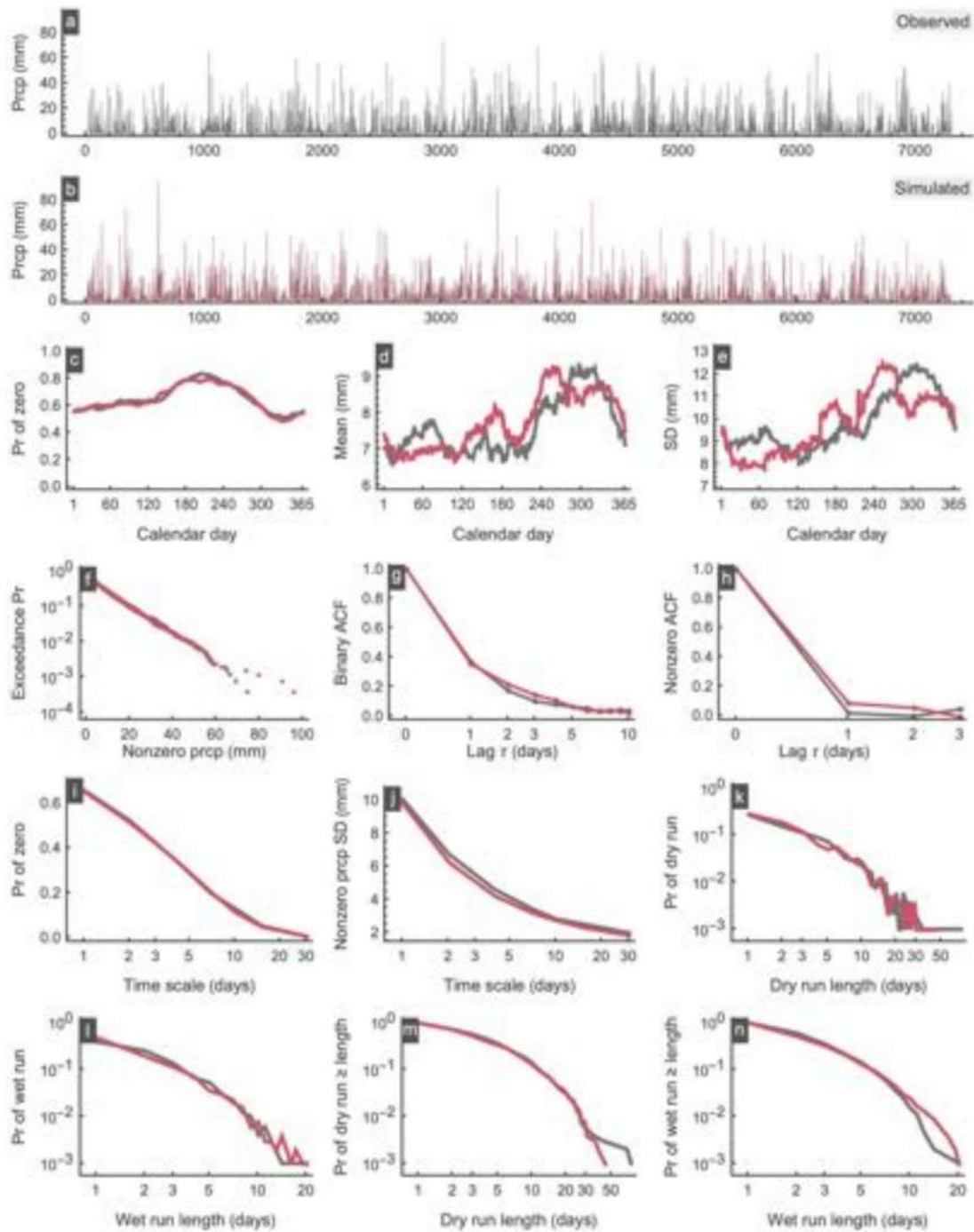


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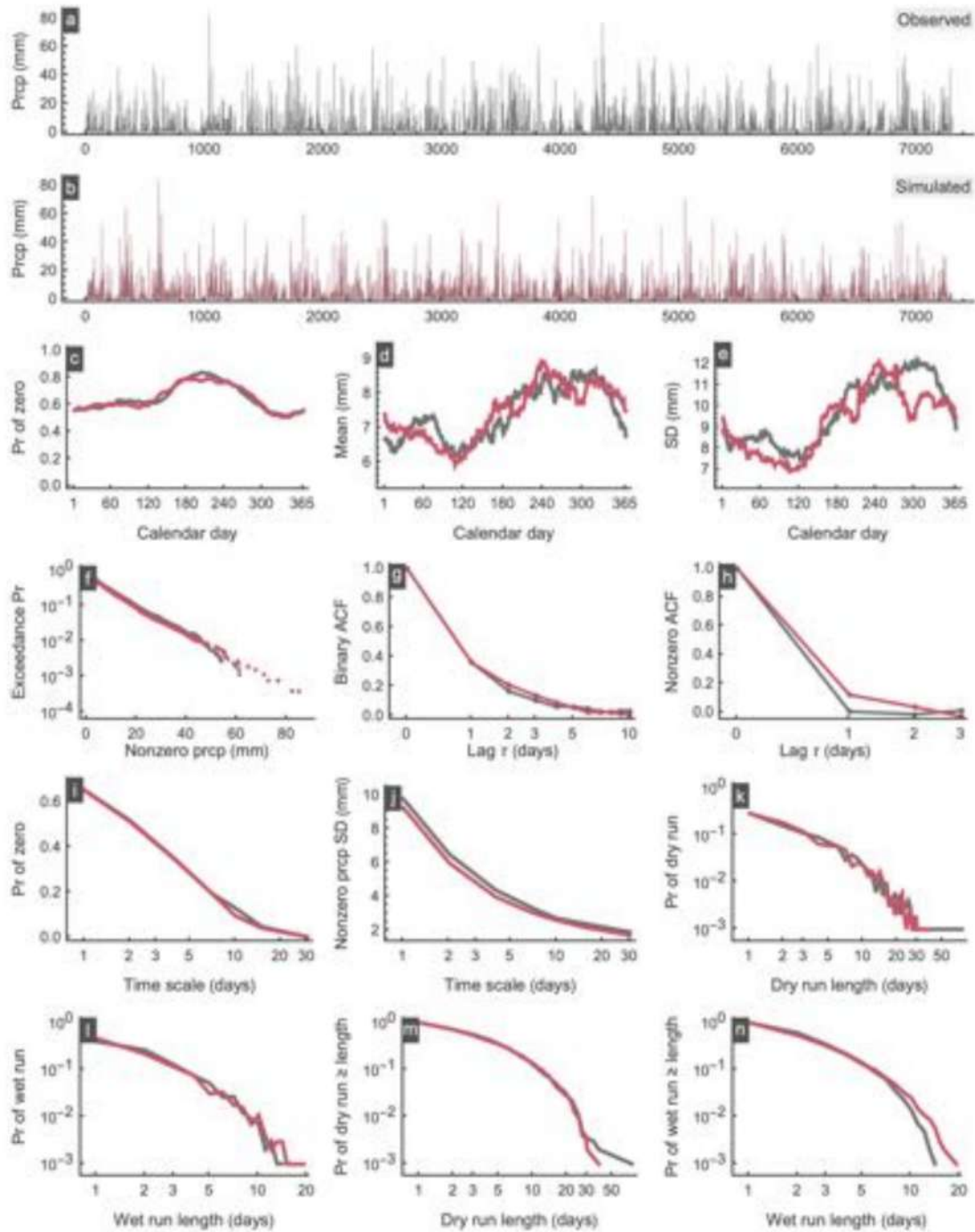


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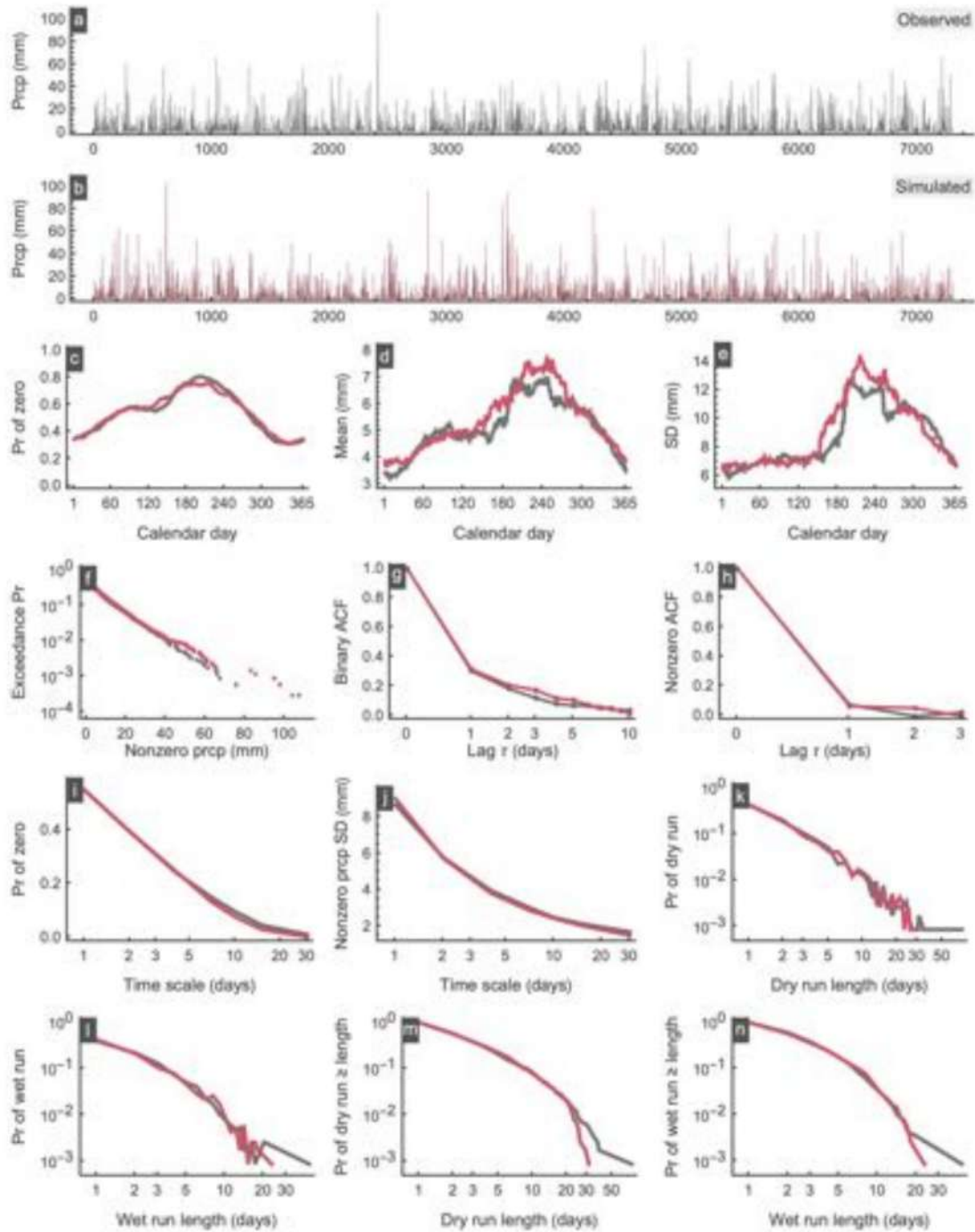


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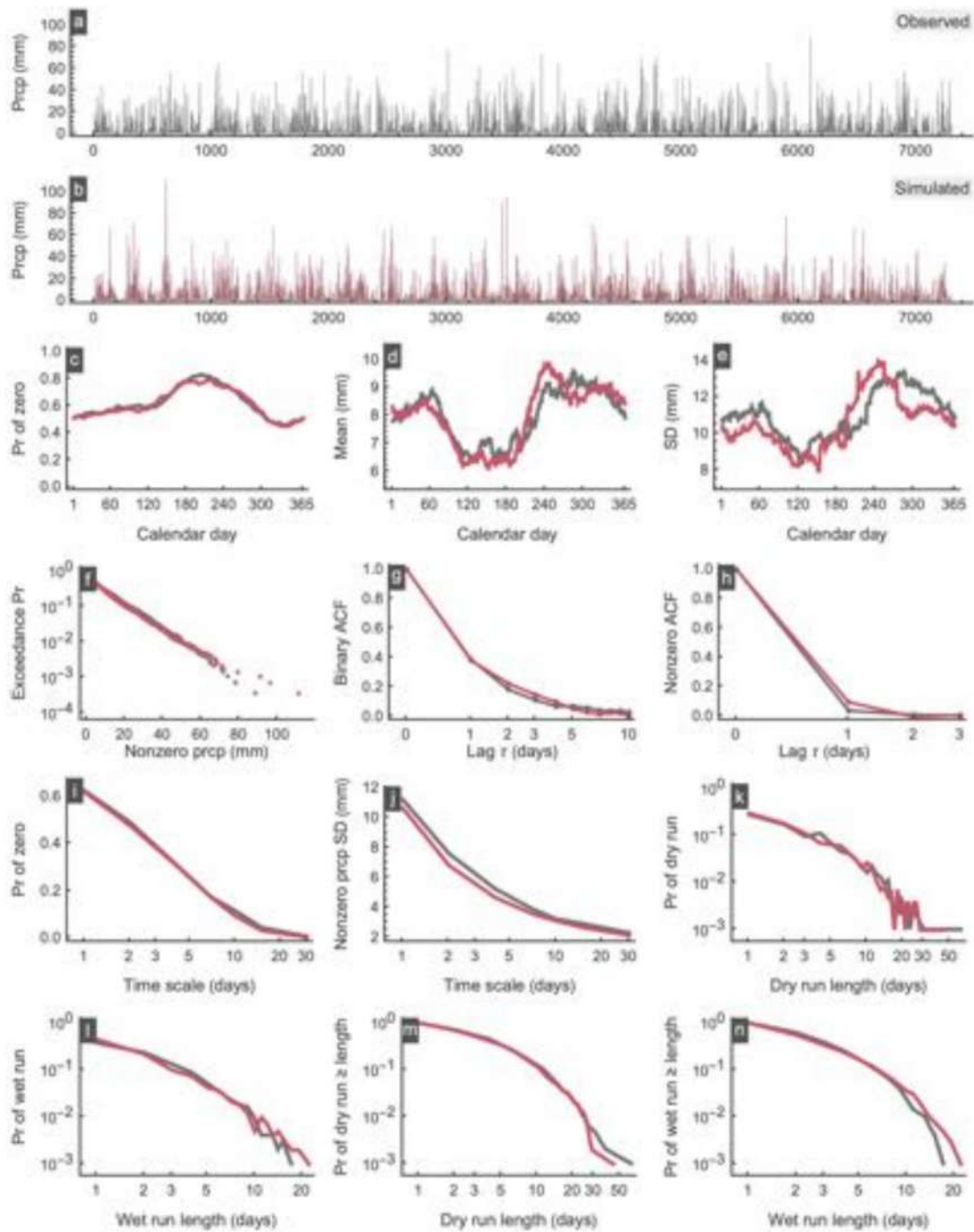


Figure A30. Simulation results for station PluvioMetro 38.

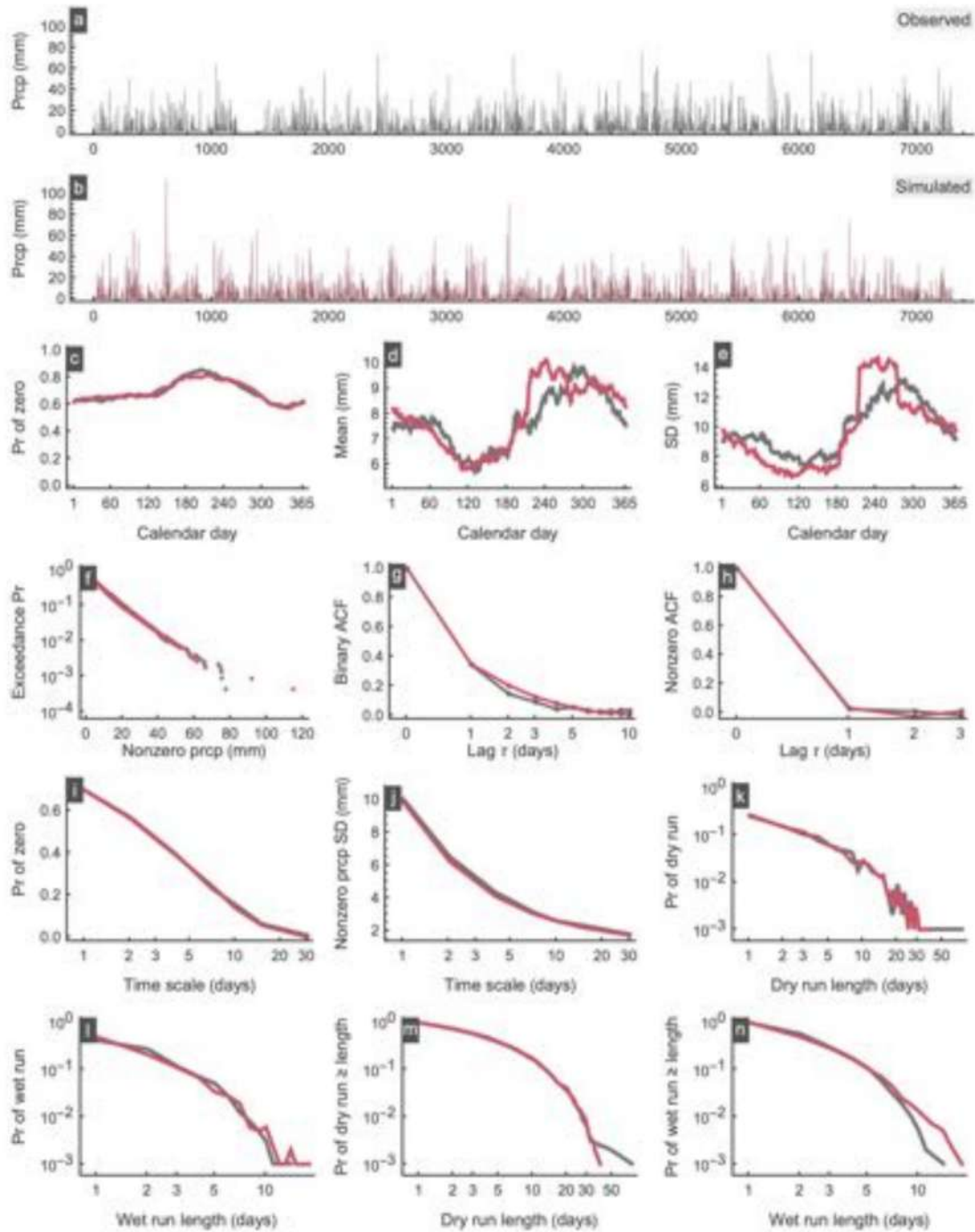


Figure A31. Simulation results for station PluvioMetro 39.

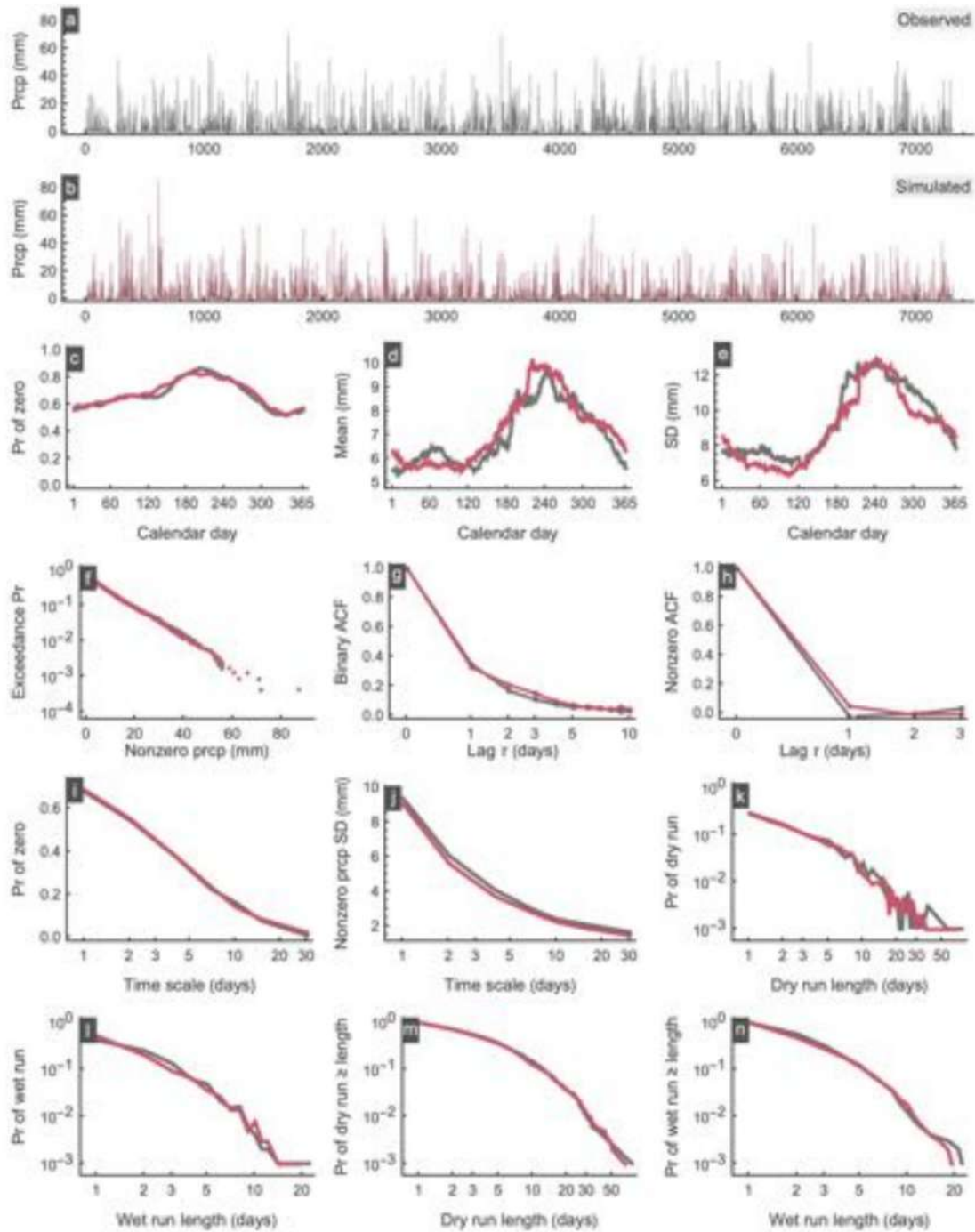


Figure A32. Simulation results for station PluvioMetro 40.

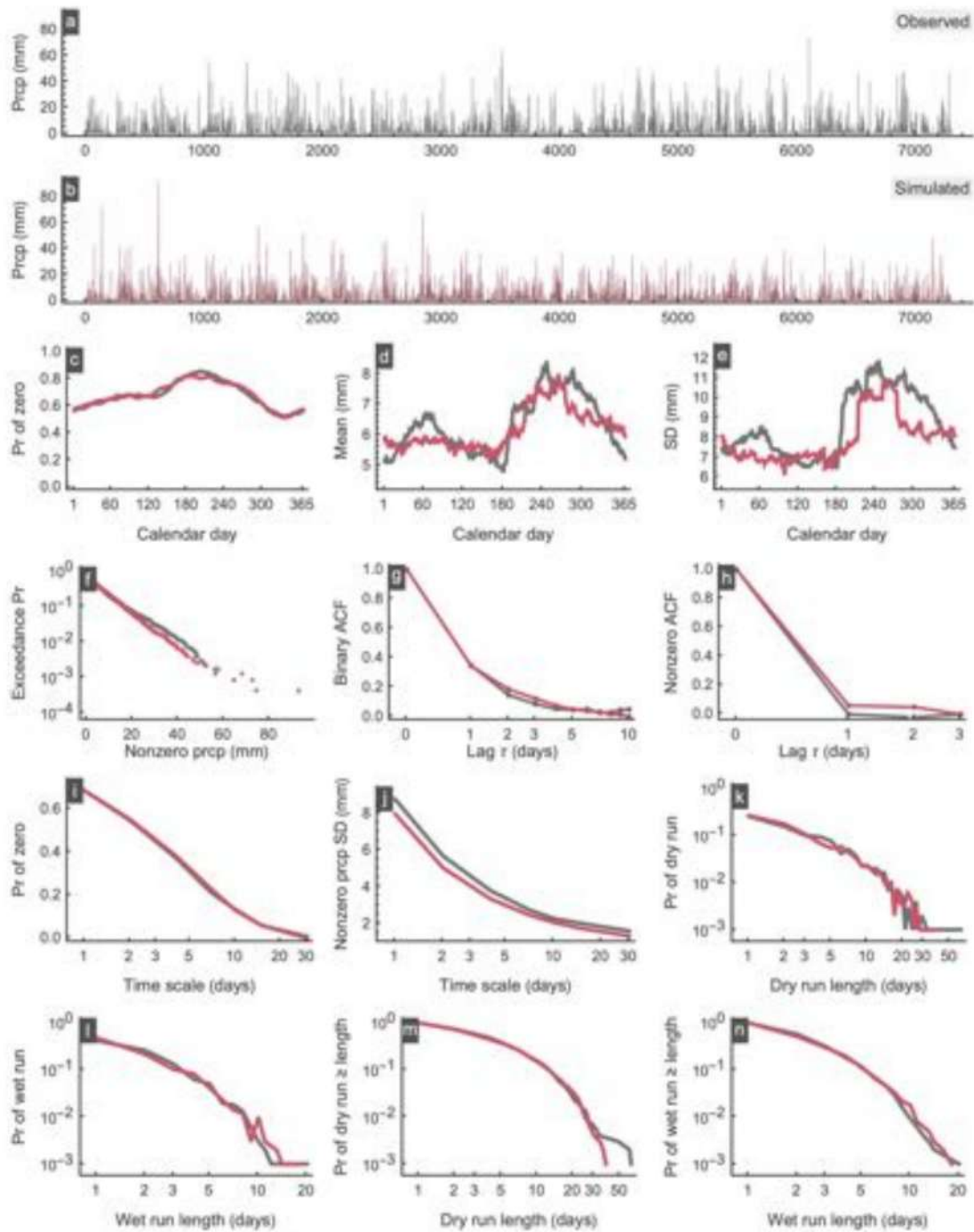


Figure A33. Simulation results for station PluvioMetro 41.

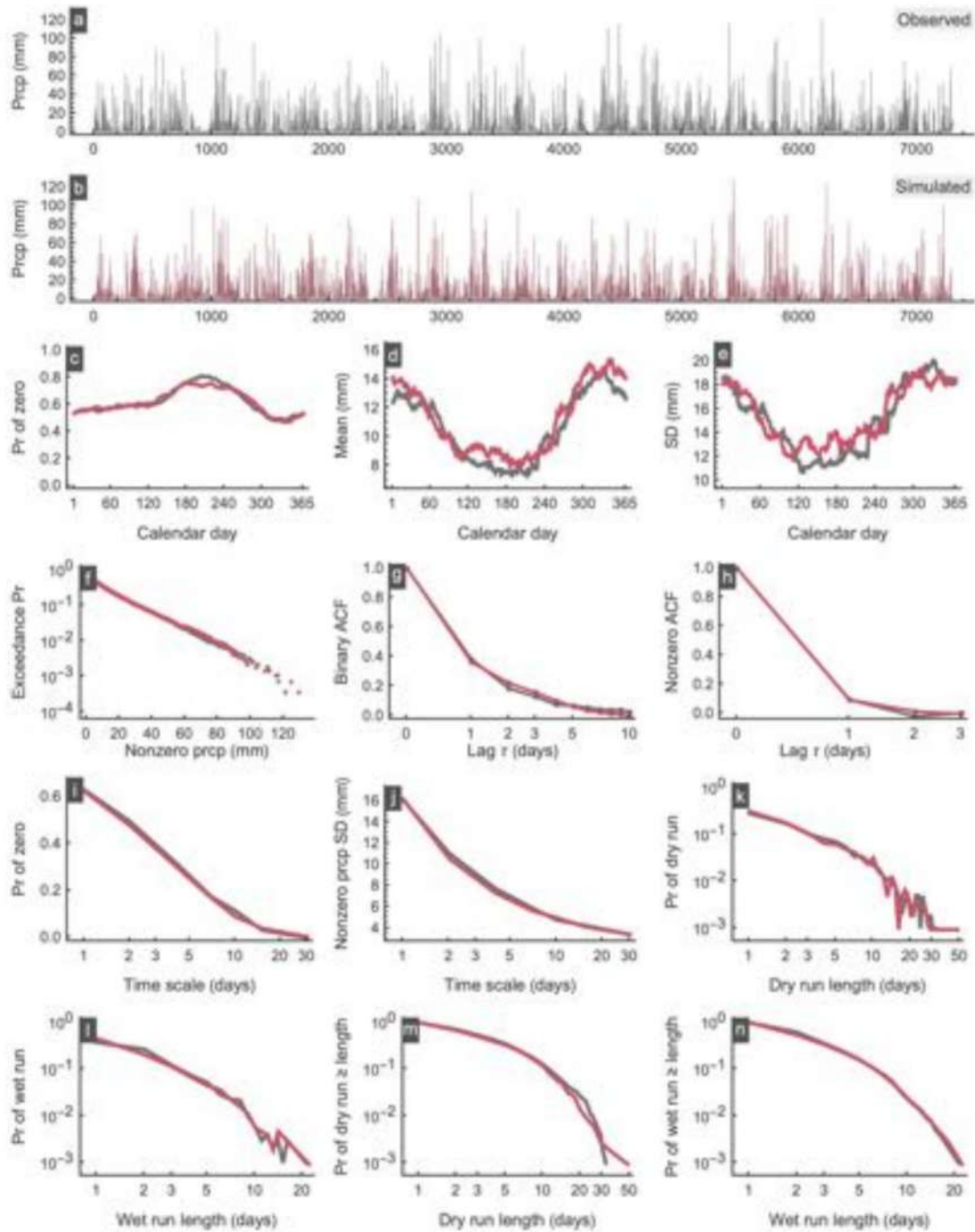


Figure A34. Simulation results for station PluvioMetro 42.

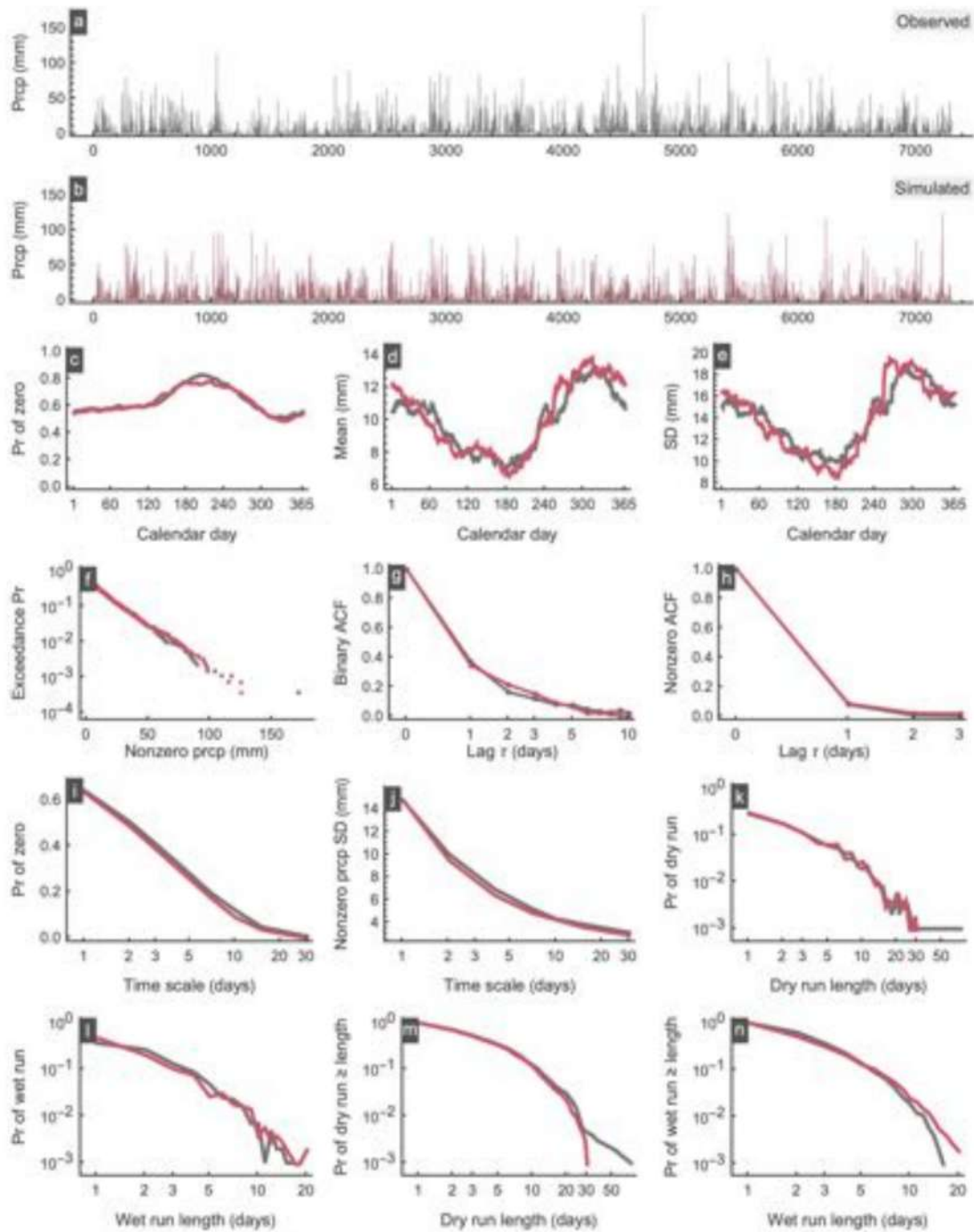


Figure A35. Simulation results for station PluvioMetro 43.

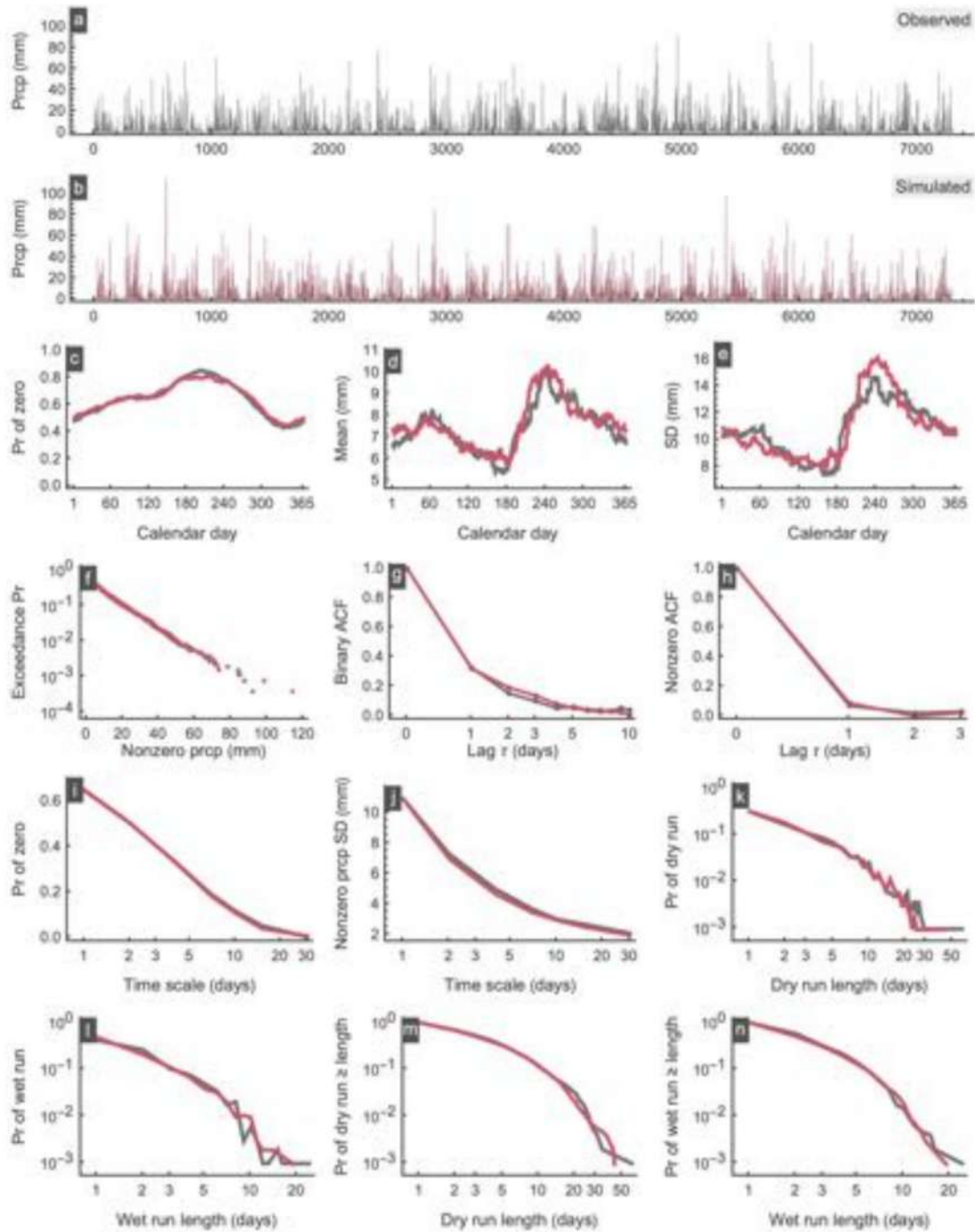


Figure A36. Simulation results for station PluvioMetro 44.

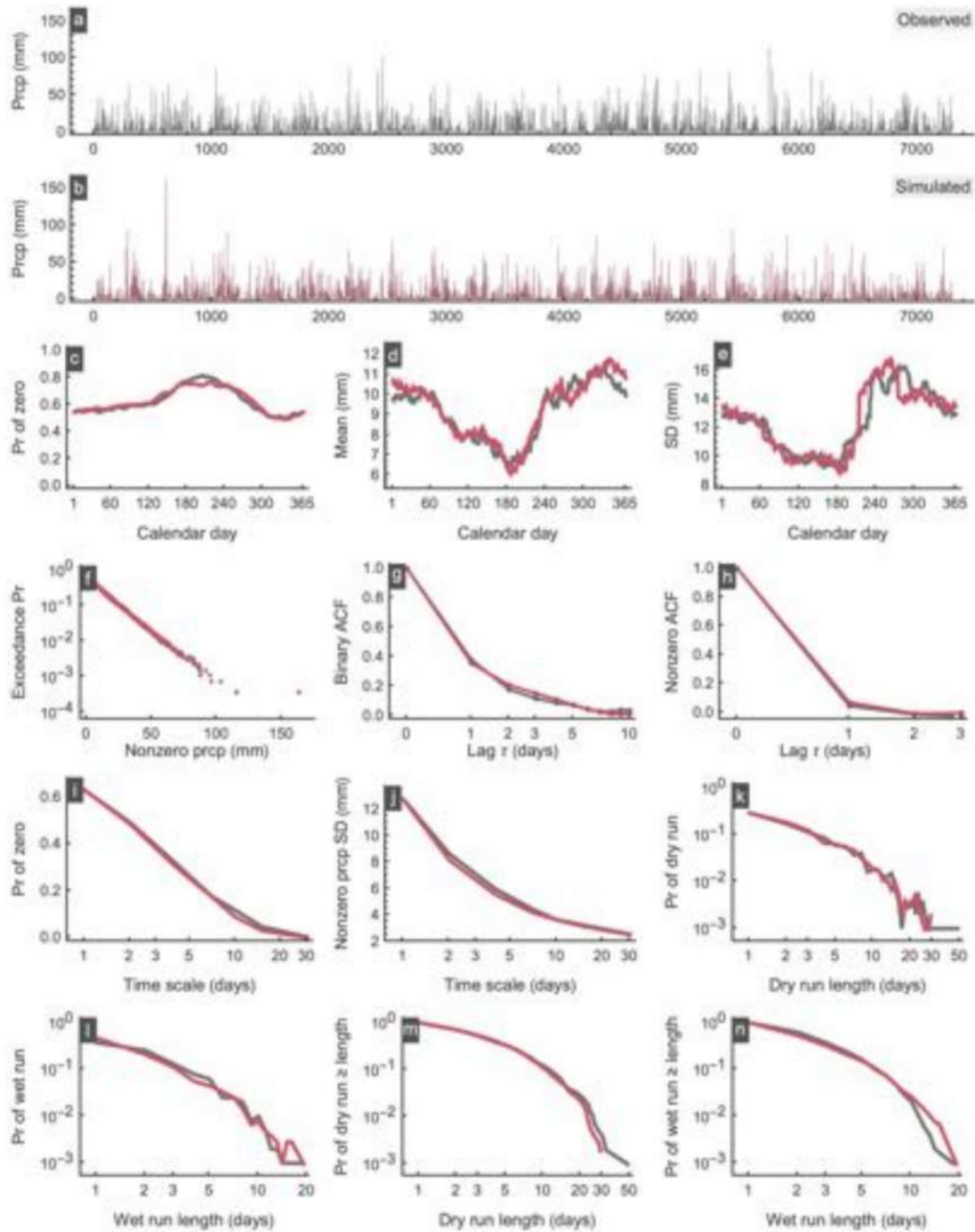


Figure A37. Simulation results for station PluvioMetro 45.

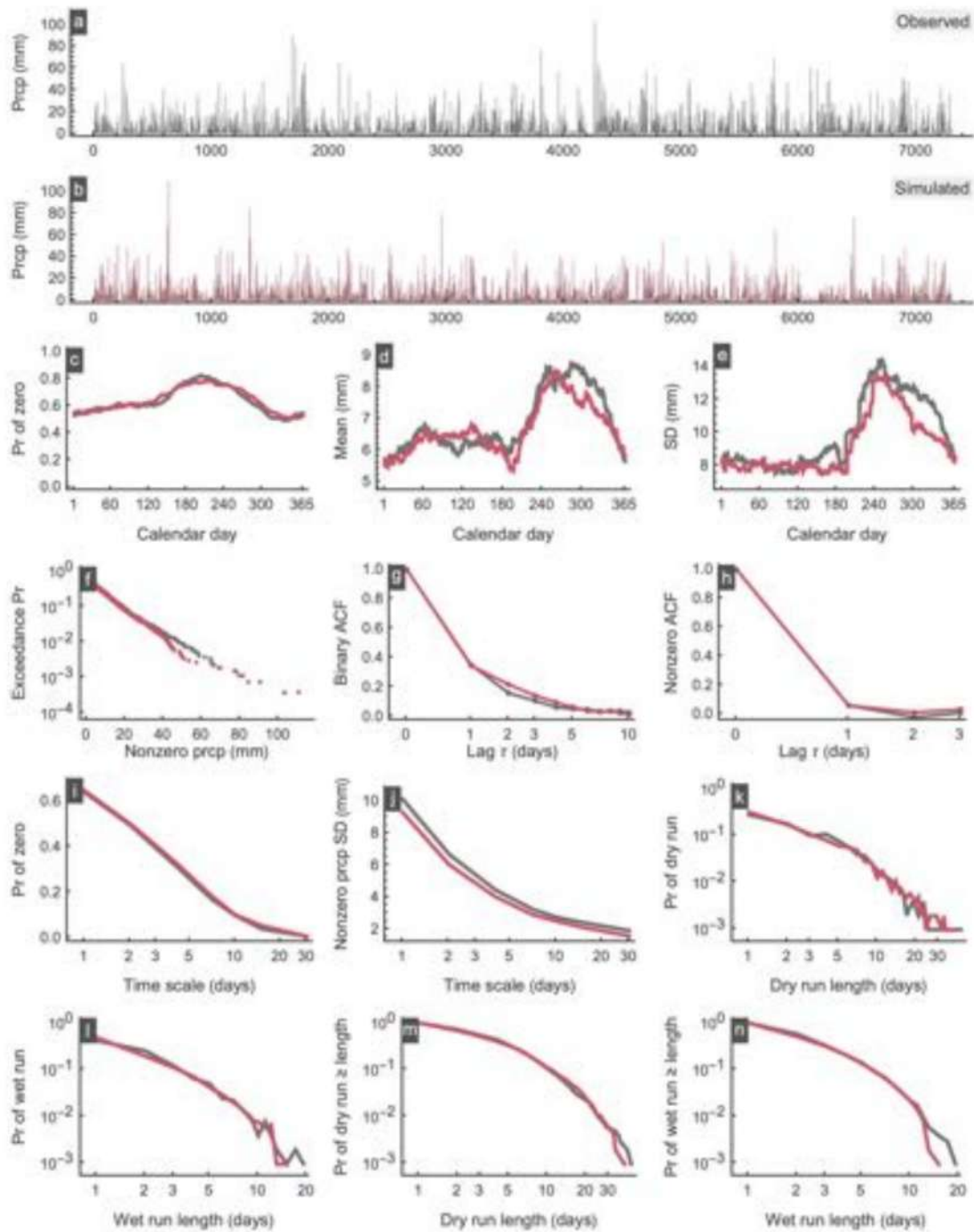


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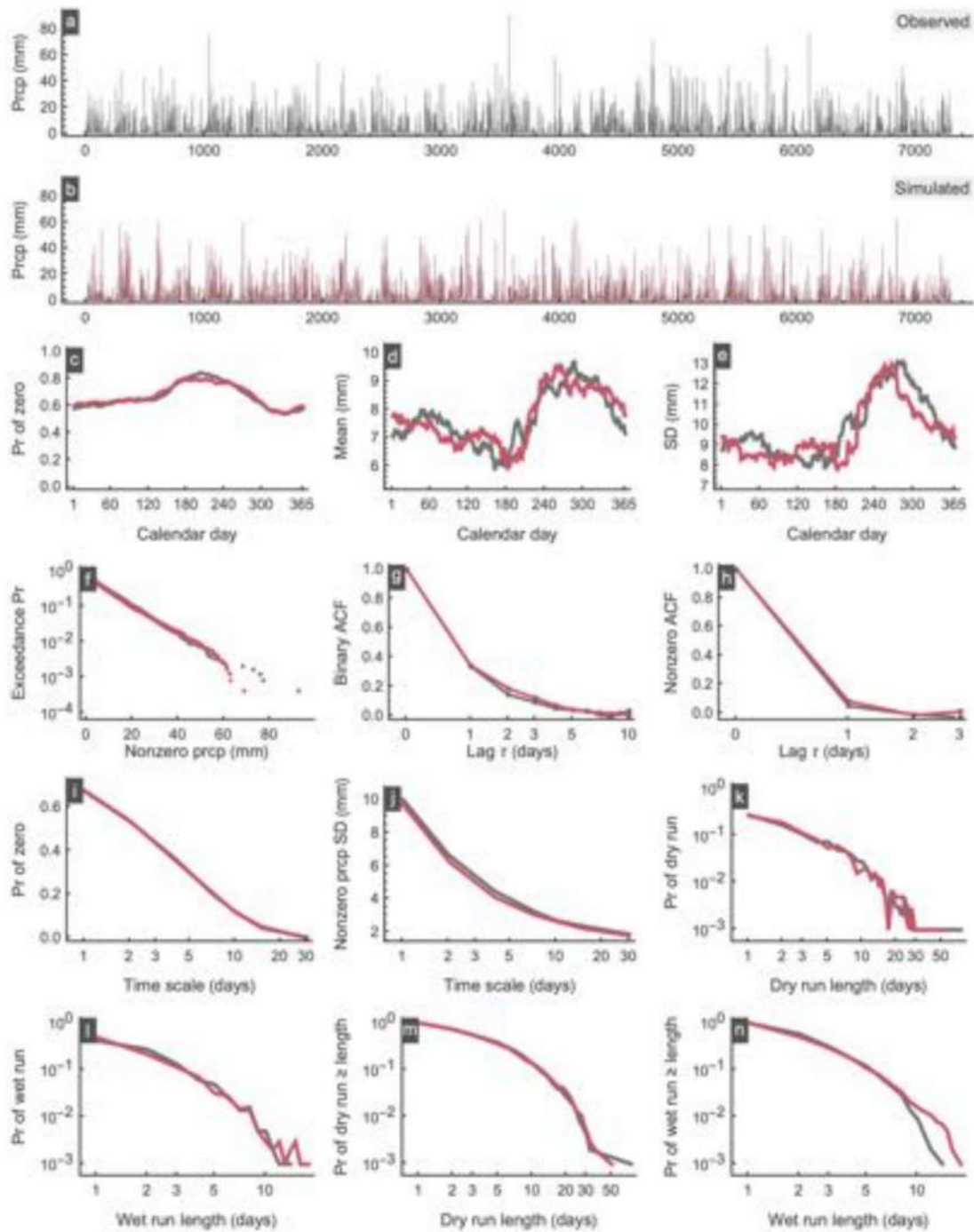


Figure A39. Simulation results for station PluvioMetro 47.

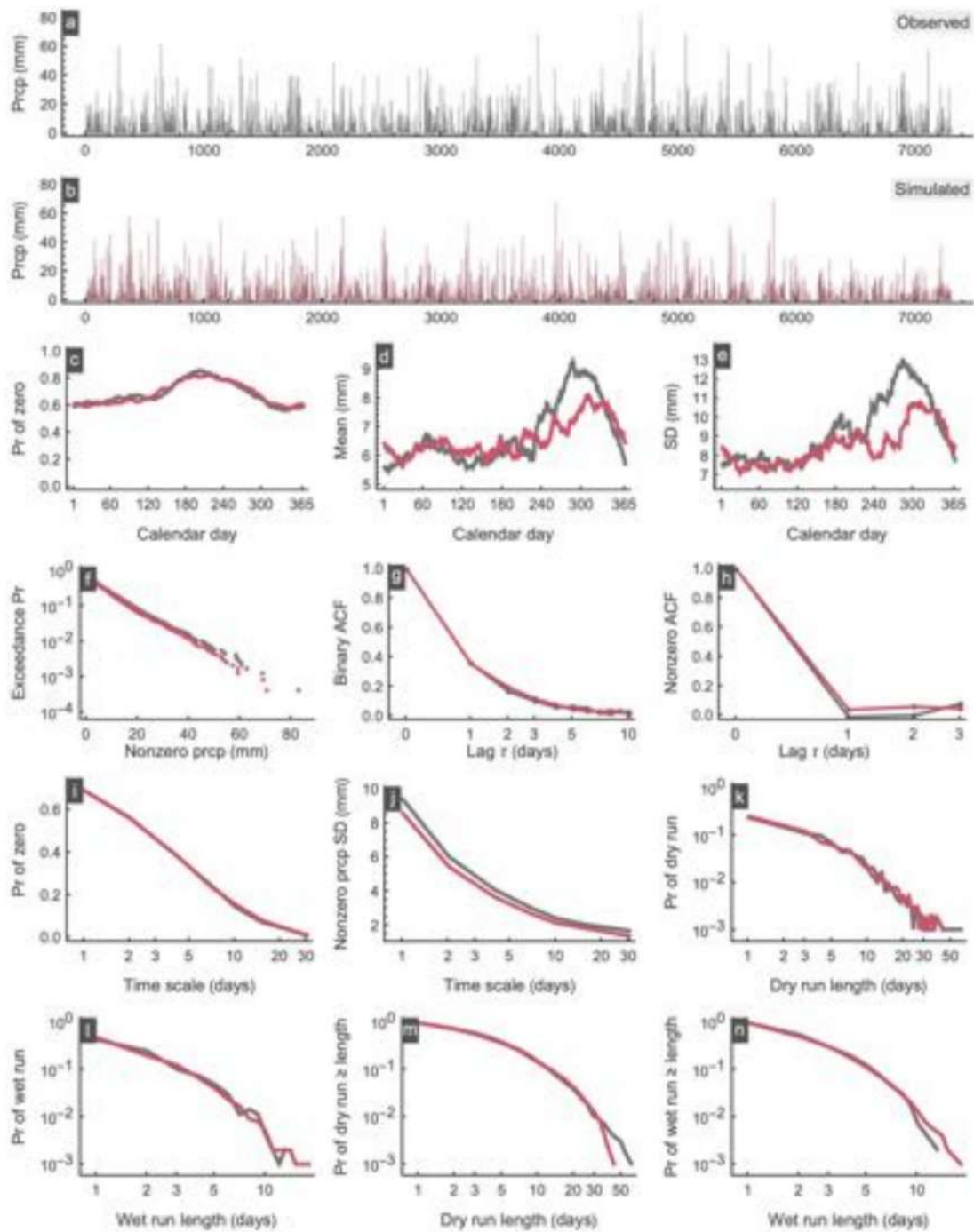


Figure A40. Simulation results for station PluvioMetro 48.

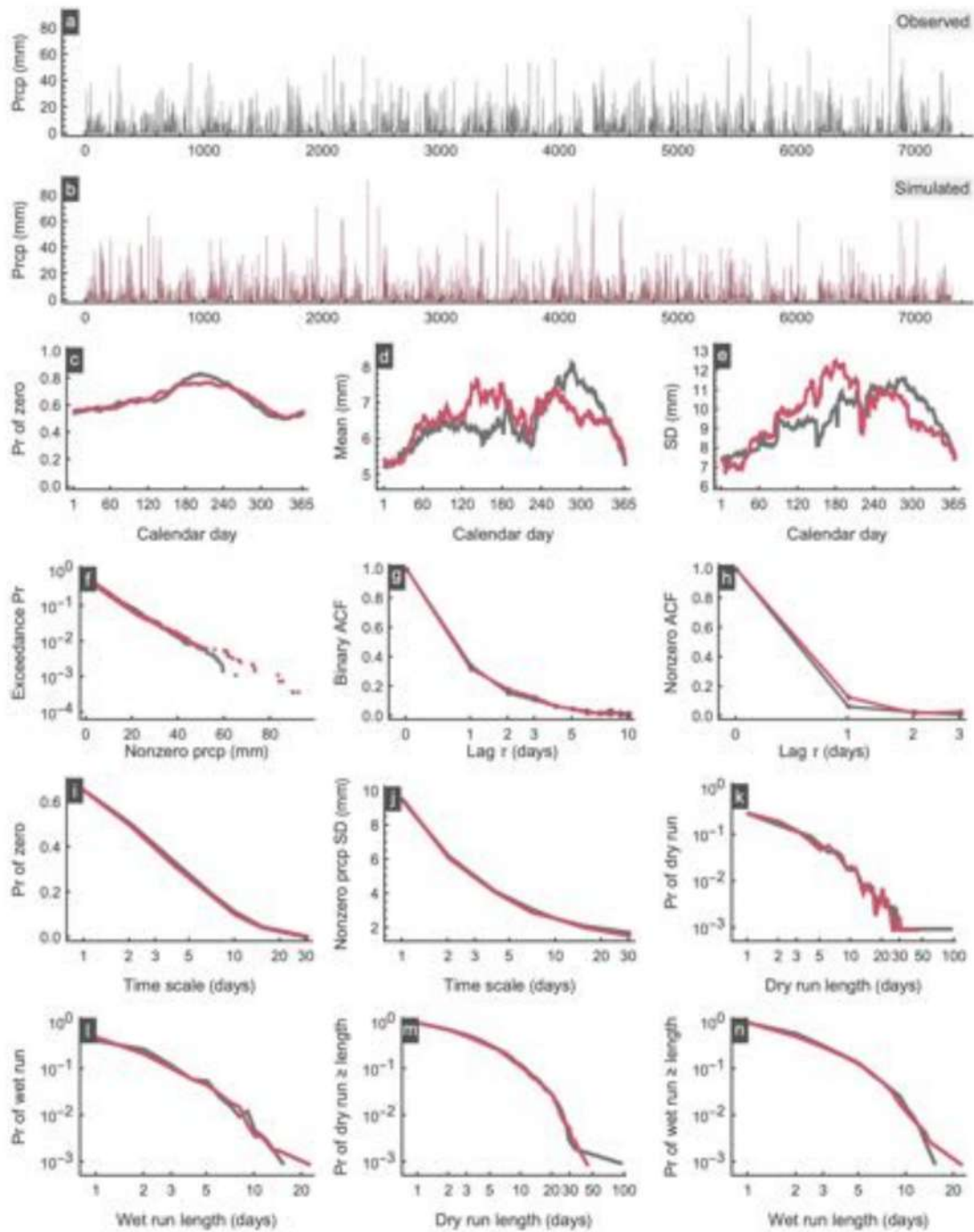


Figure A41. Simulation results for station PluvioMetro 49.

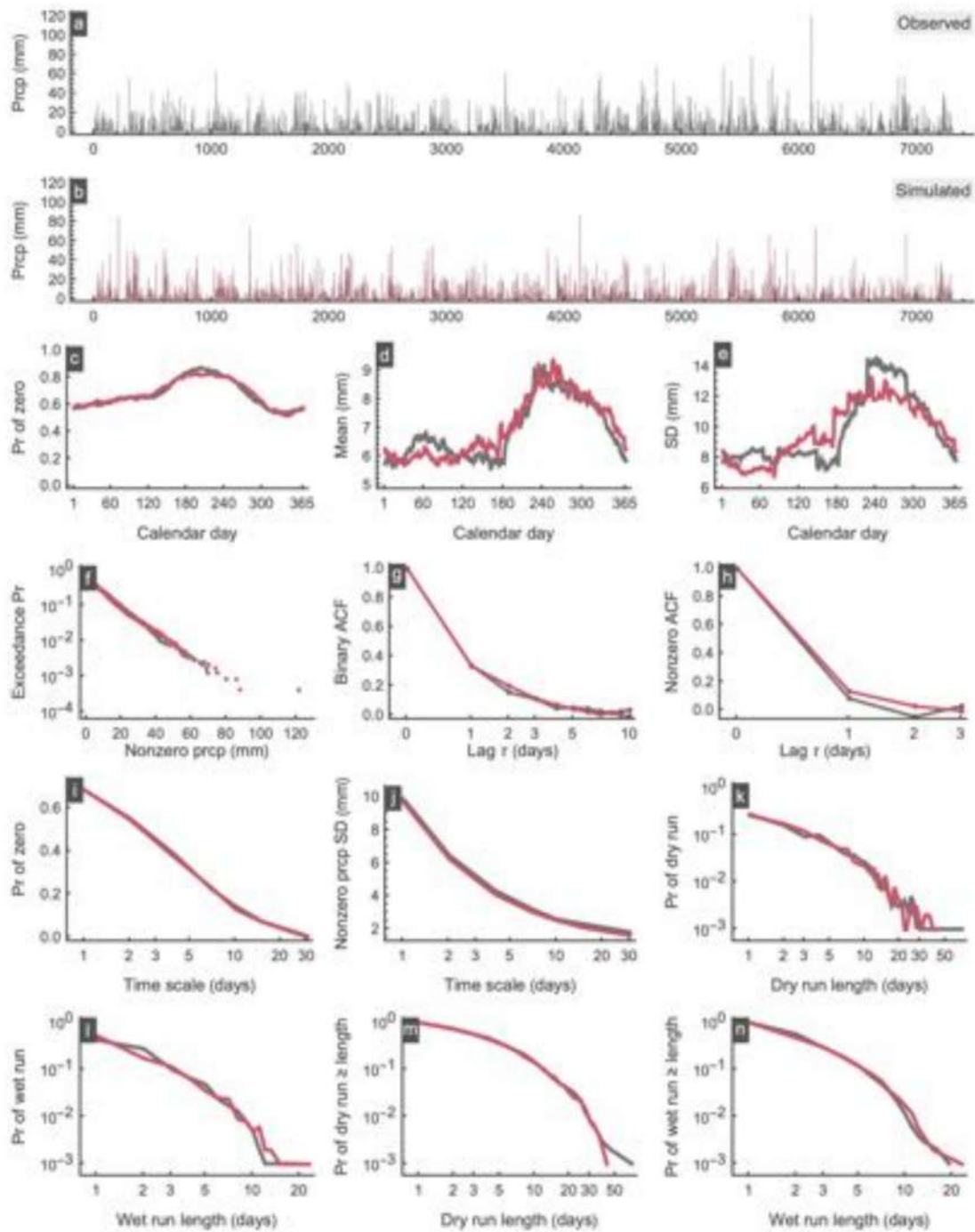


Figure A42. Simulation results for station PluvioMetro 50.

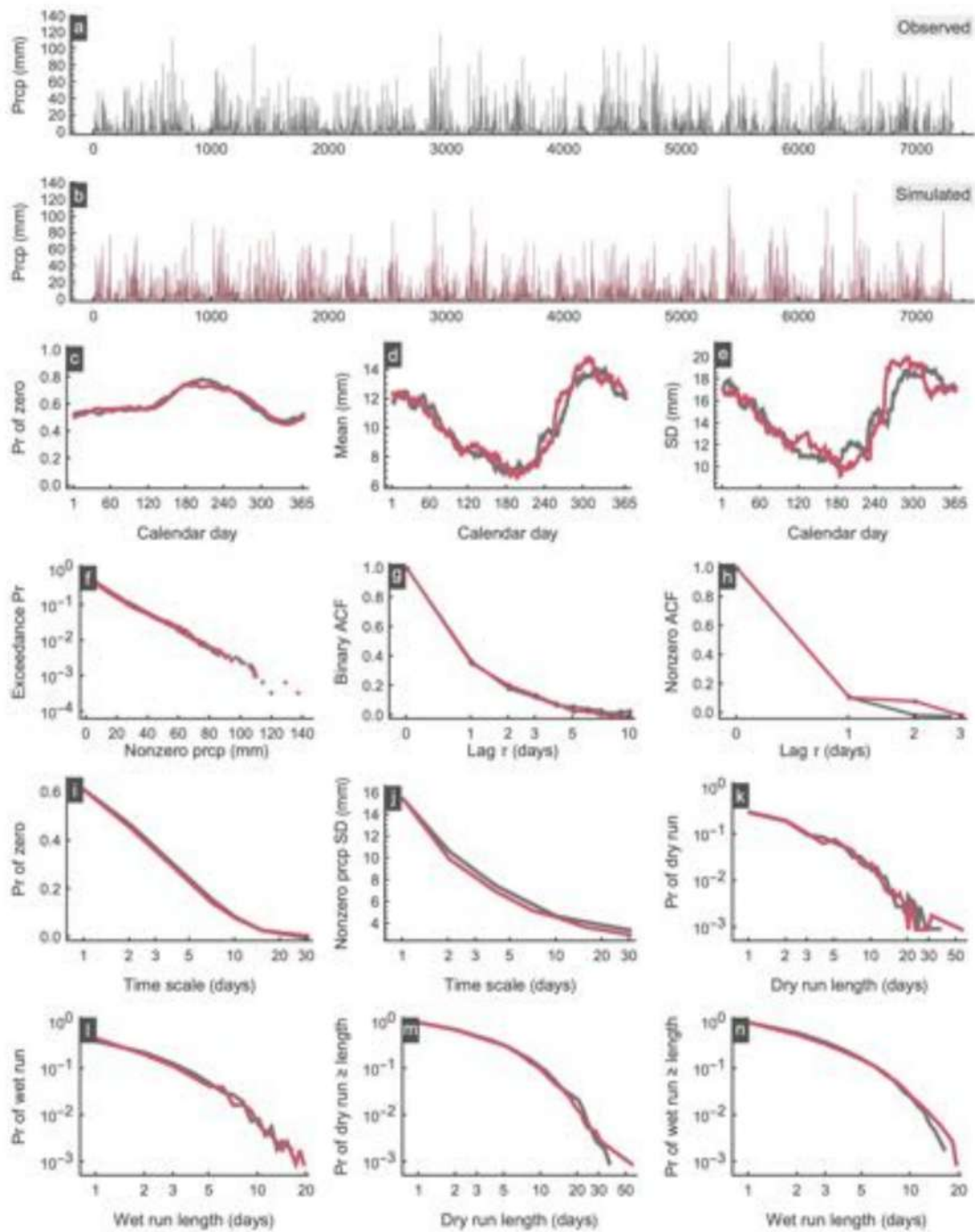


Figure A43. Simulation results for station PluvioMetro 51.

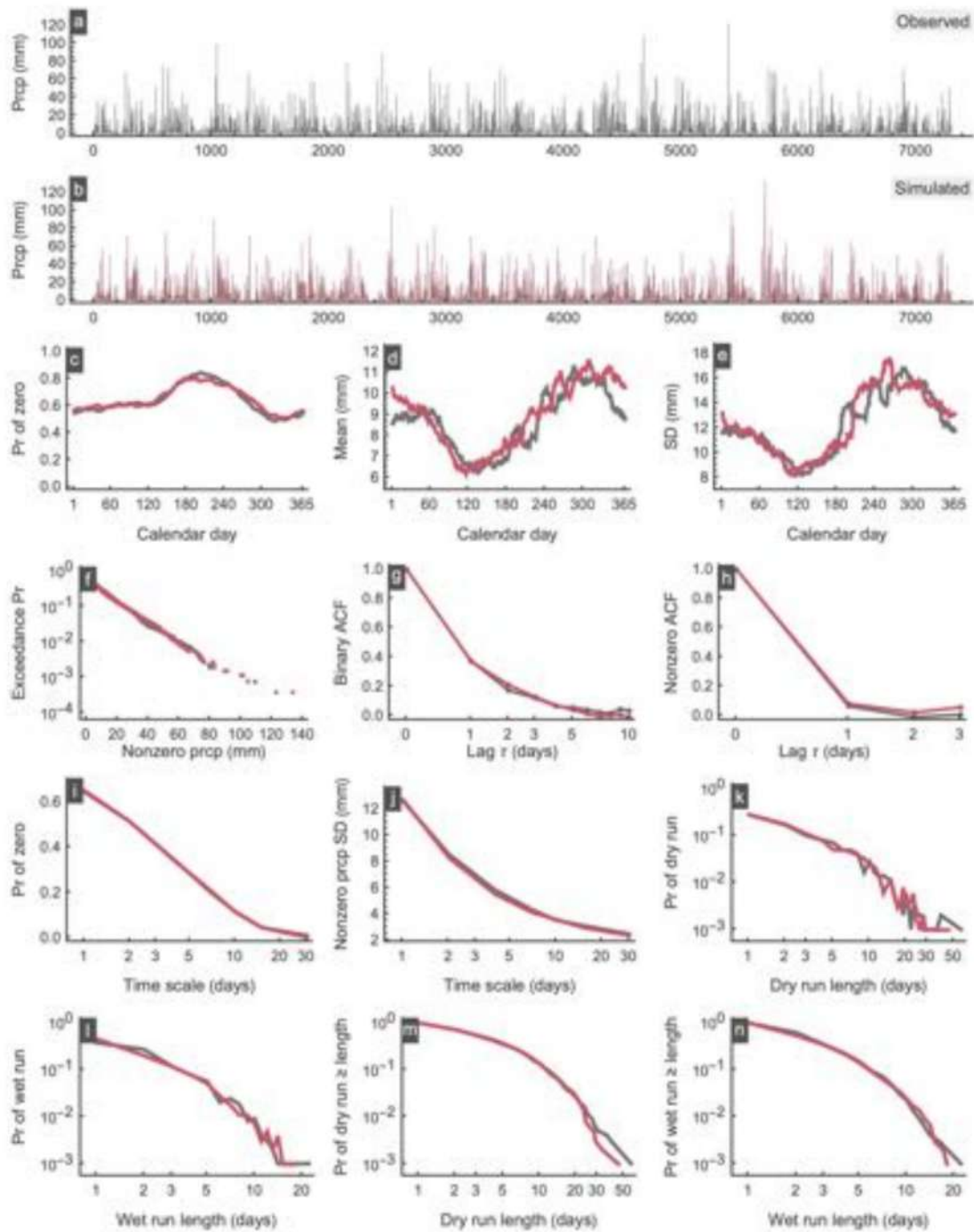


Figure A44. Simulation results for station PluvioMetro 52.

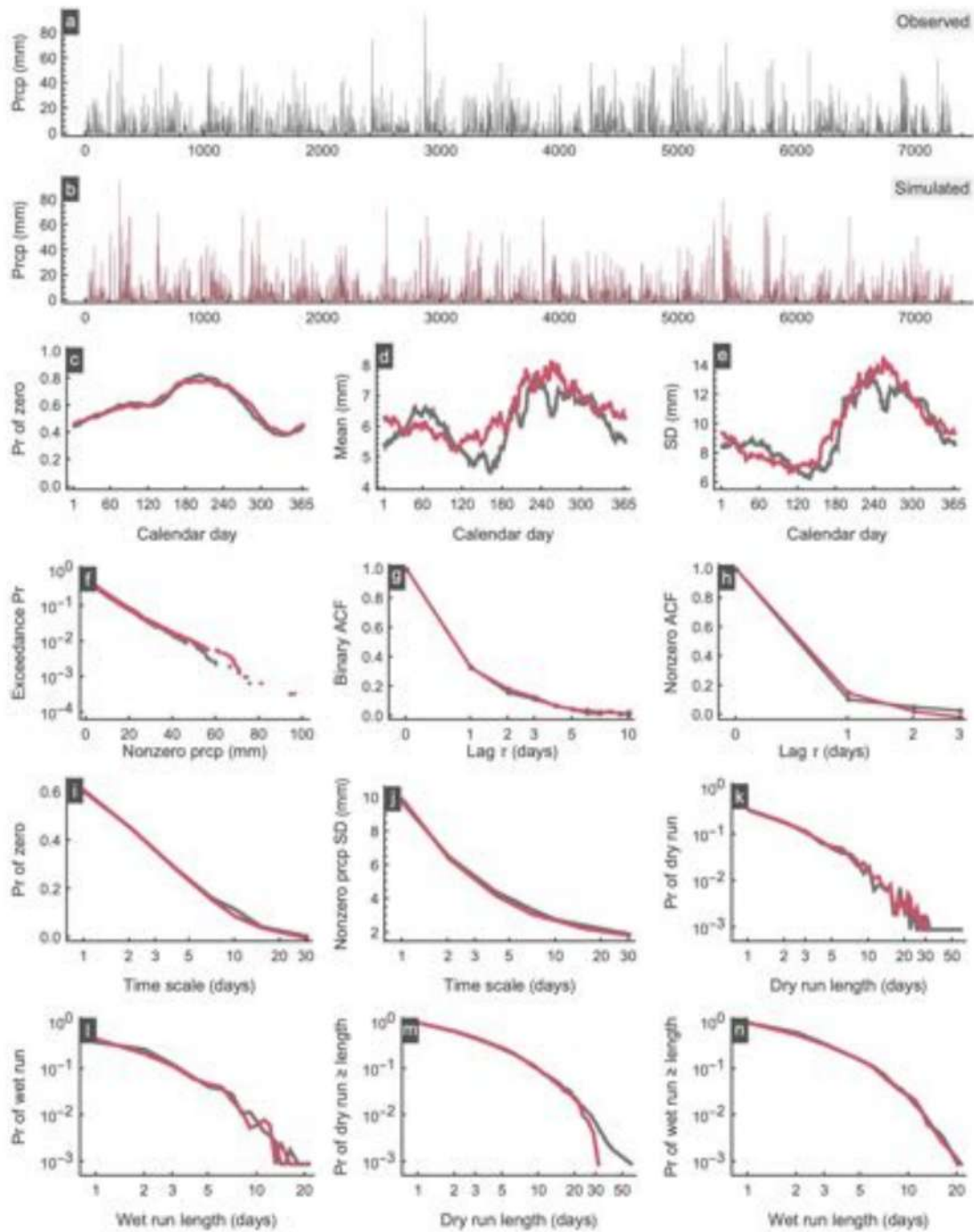


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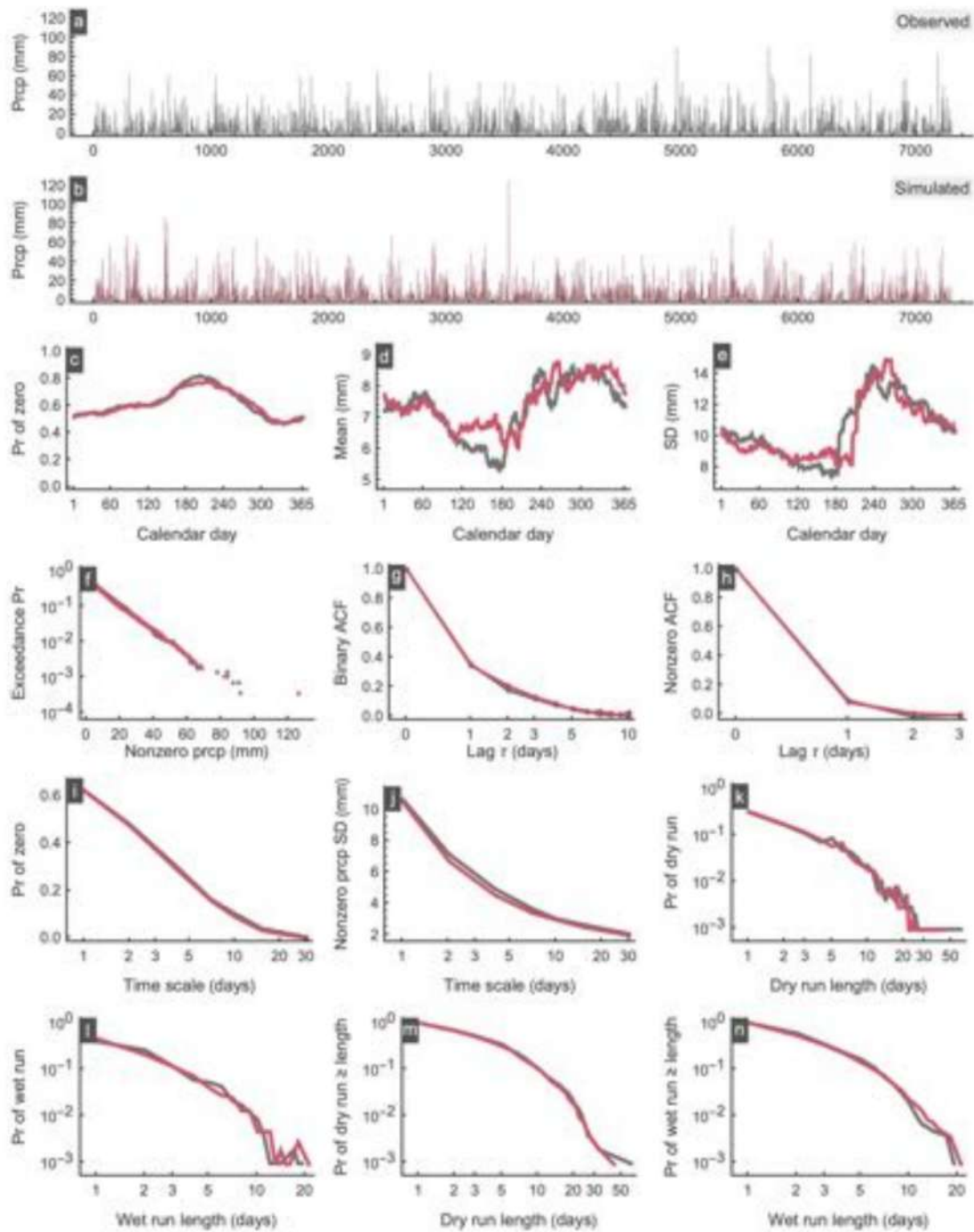


Figure A46. Simulation results for station PluvioMetro 54.

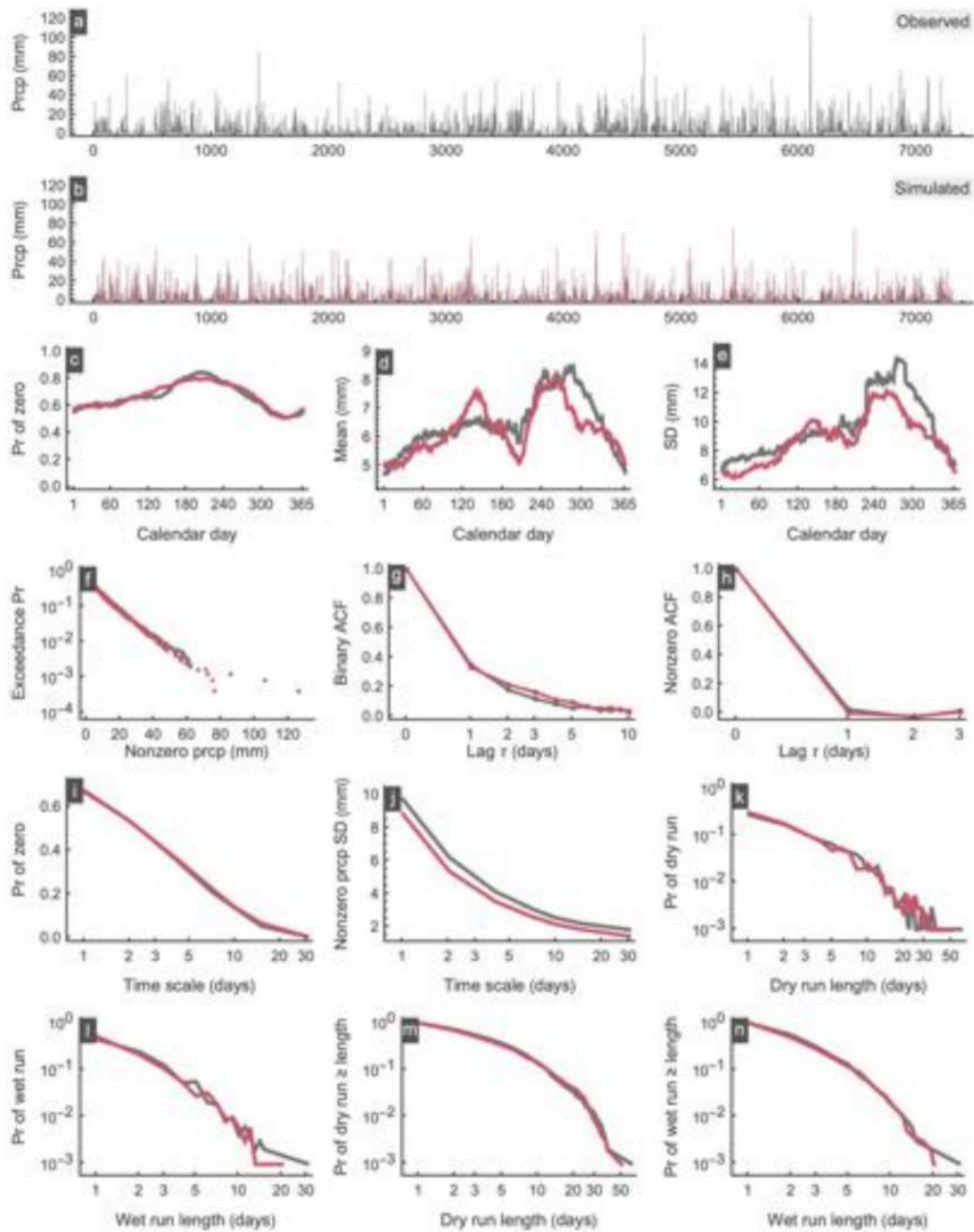


Figure A47. Simulation results for station PluvioMetro 55.

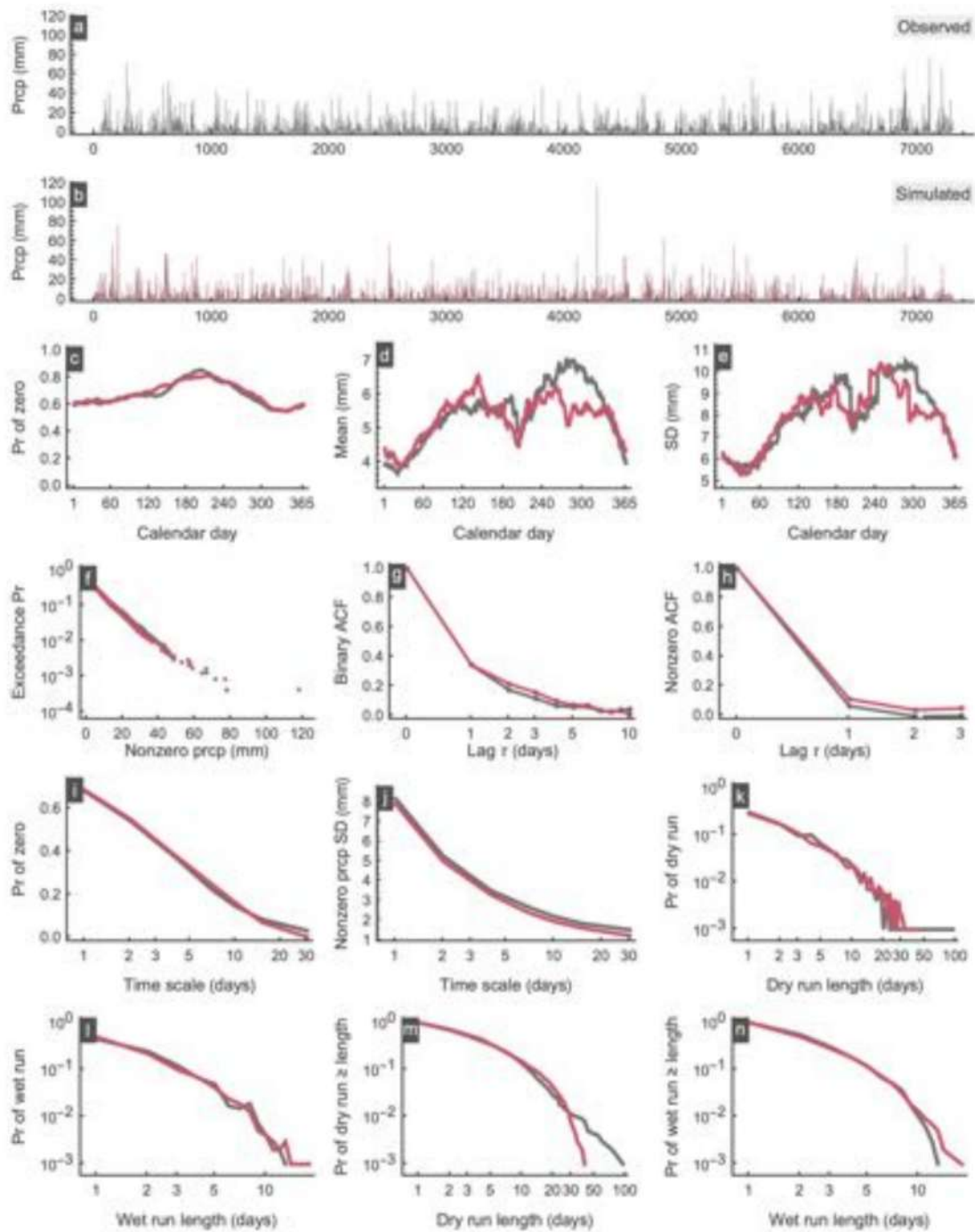


Figure A48. Simulation results for station PluvioMetro 56.

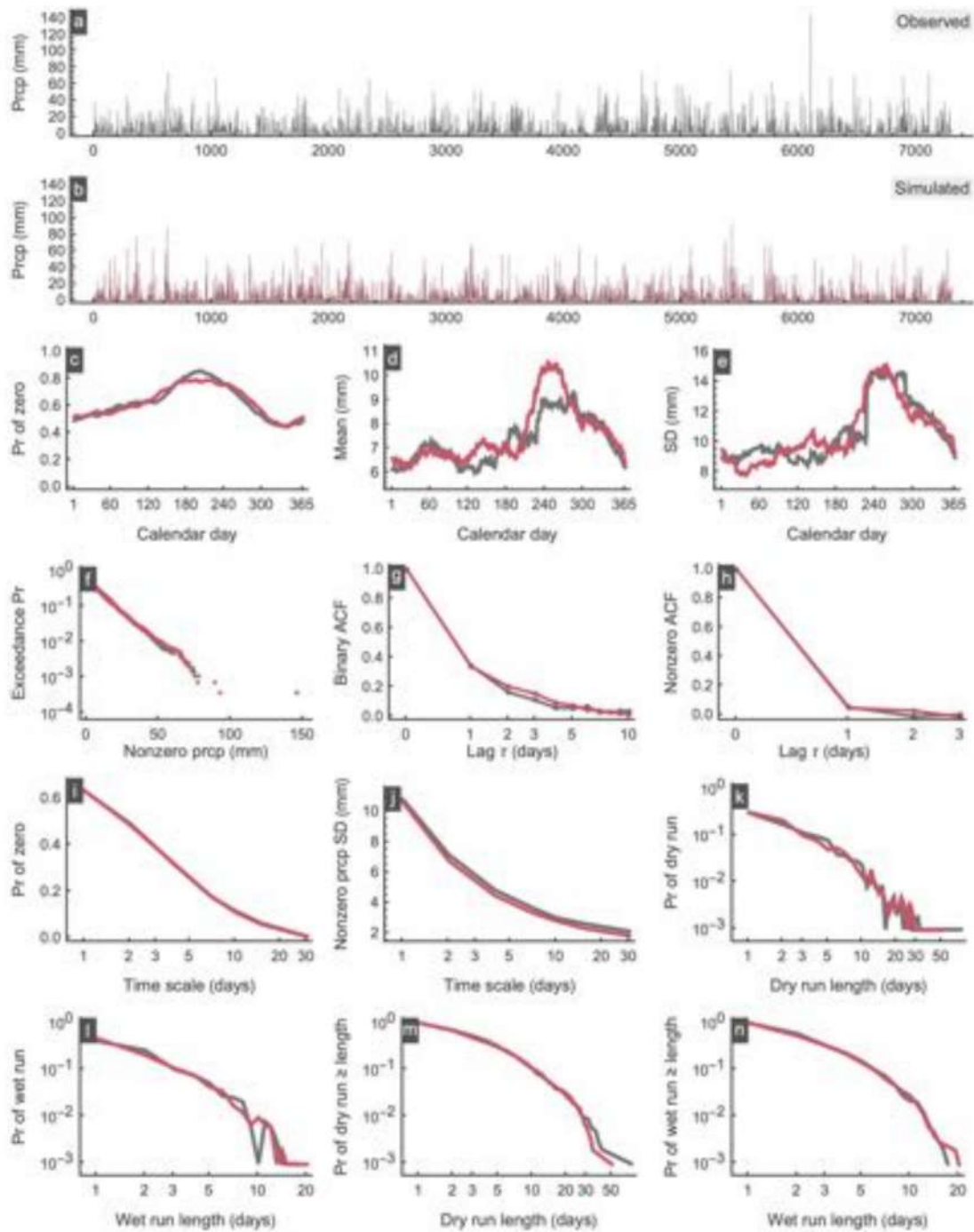


Figure A49. Simulation results for station PluvioMetro 57.

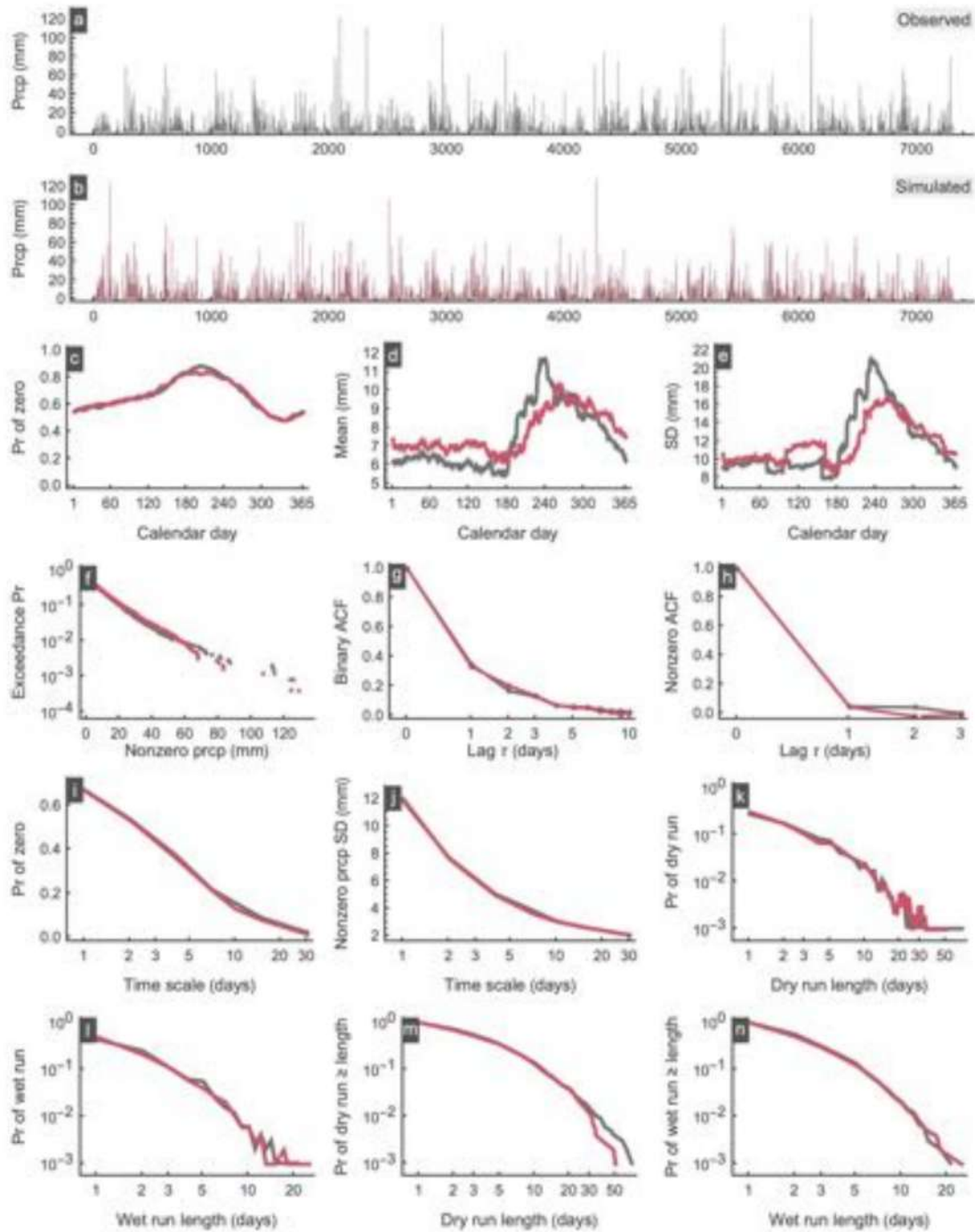


Figure A50. Simulation results for station PluvioMetro 58.

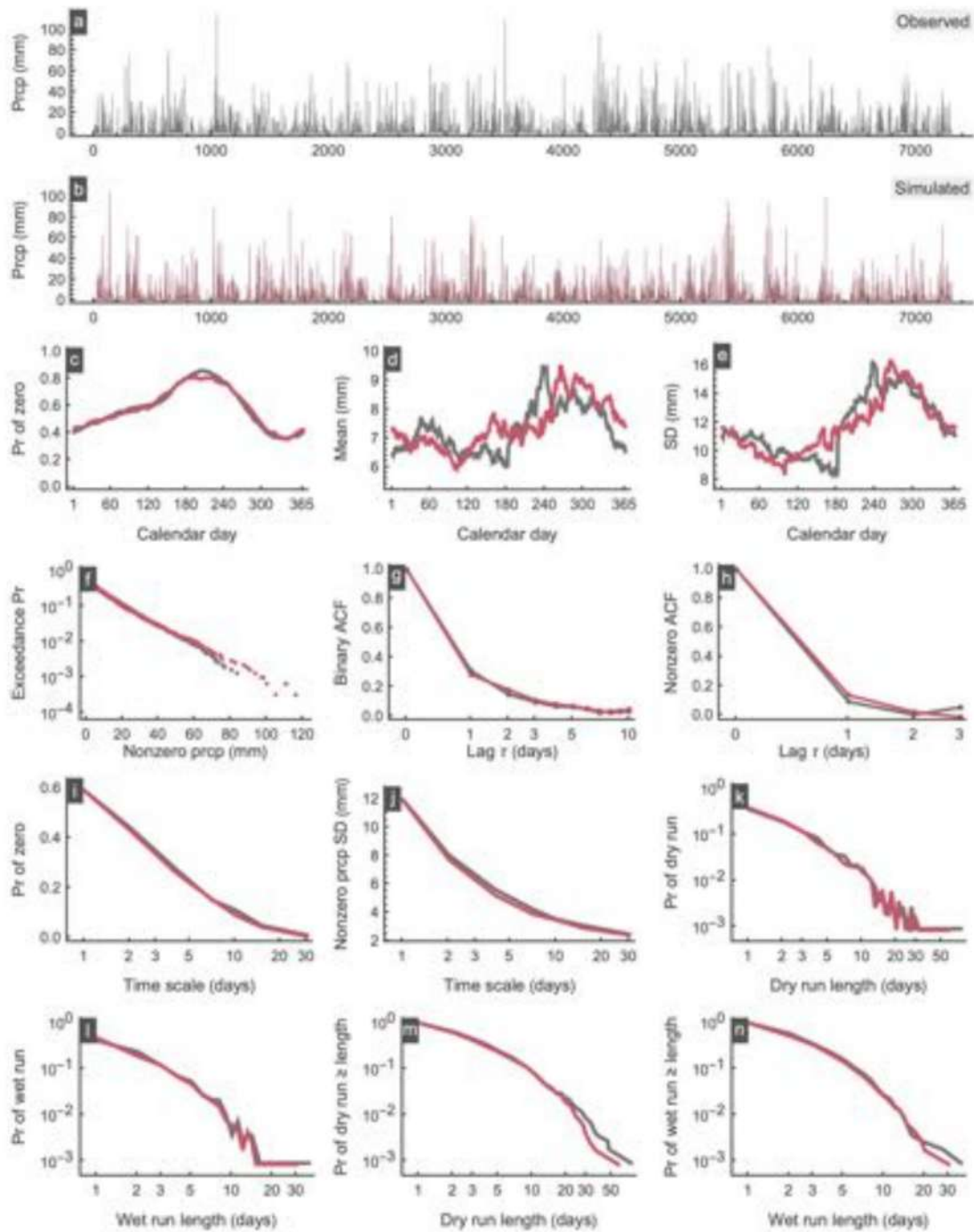


Figure A51. Simulation results for station PluvioMetro 59.

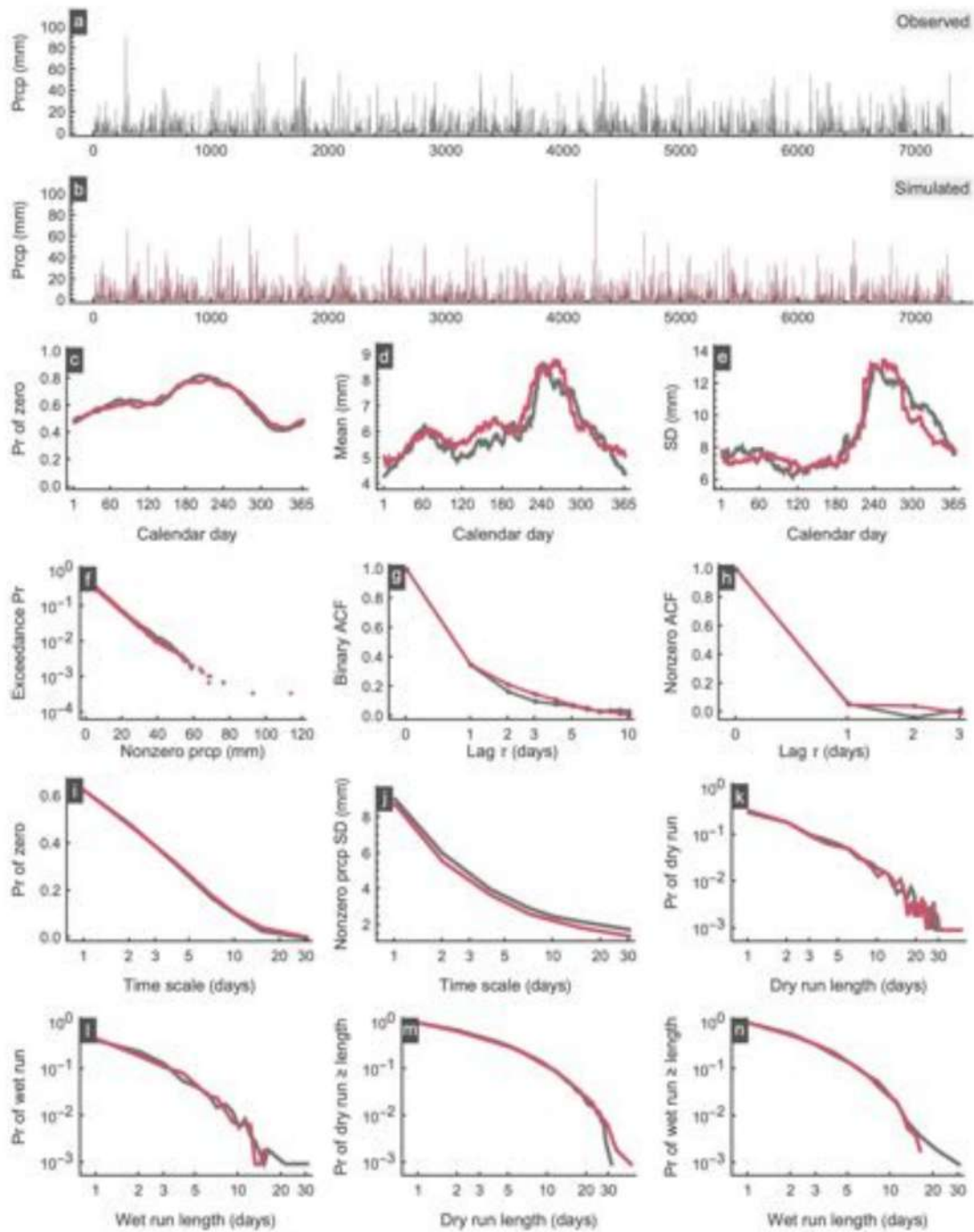


Figure A52. Simulation results for station PluvioMetro 60.

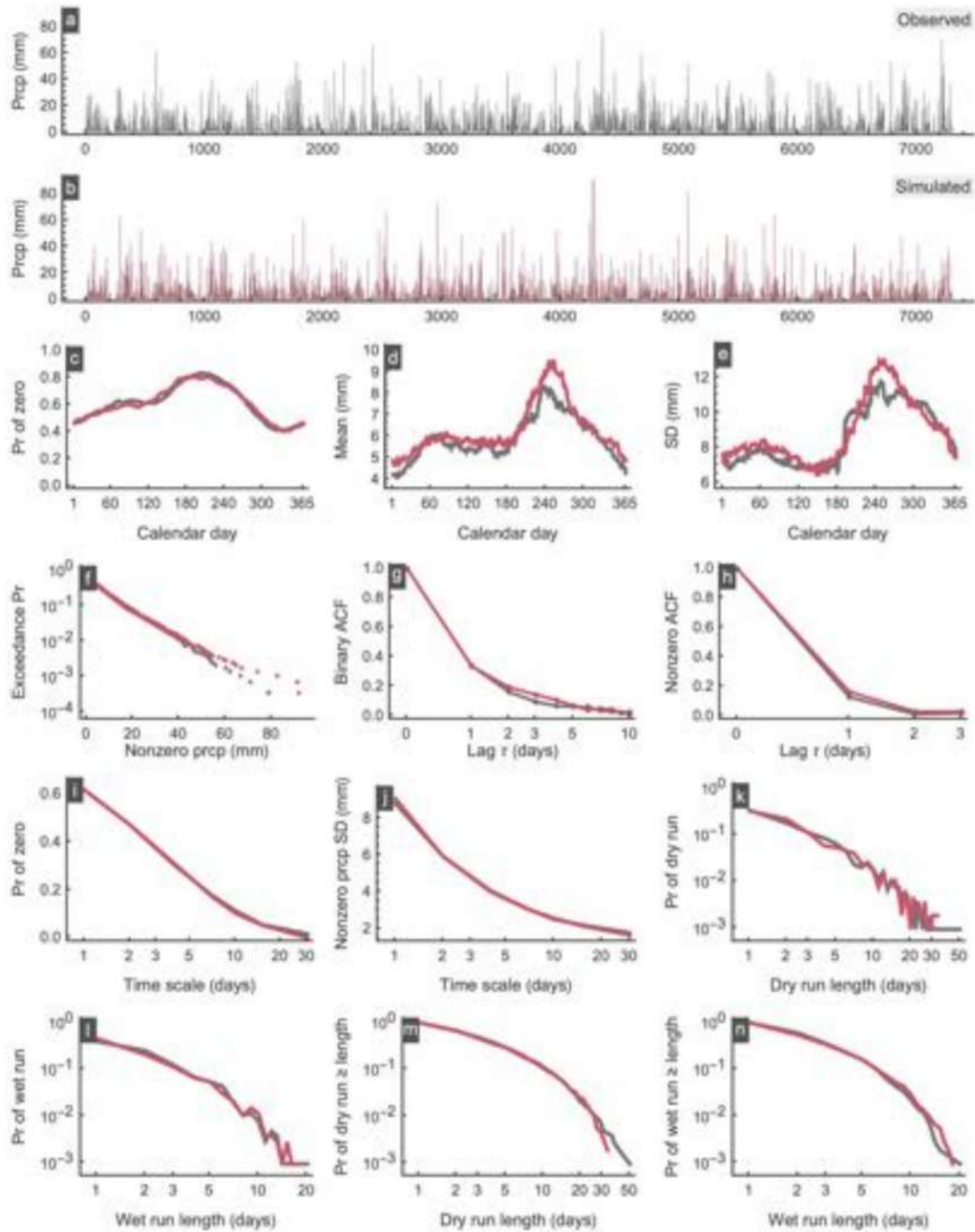


Figure A53. Simulation results for station PluvioMetro 61.

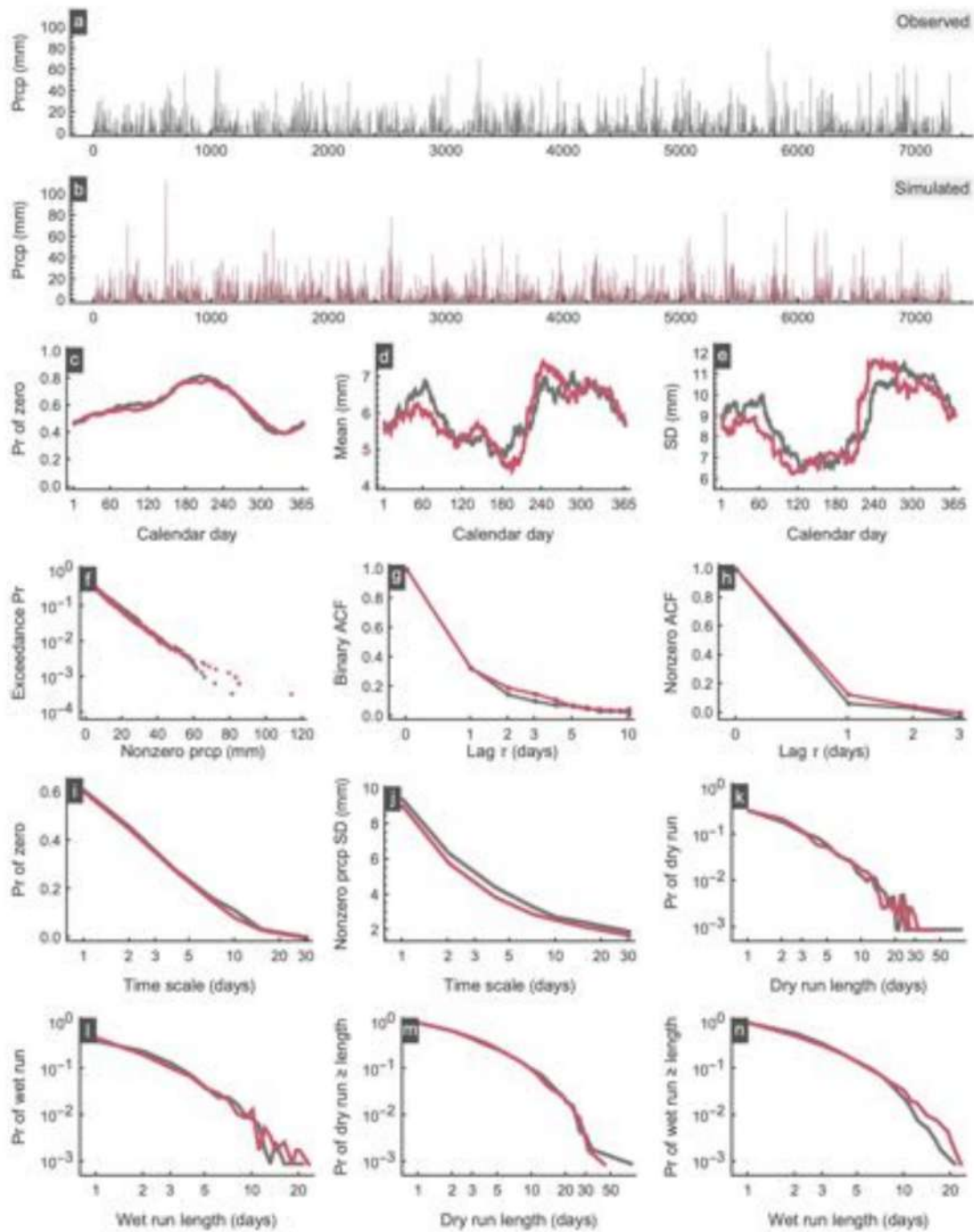


Figure A54. Simulation results for station PluvioMetro 62.

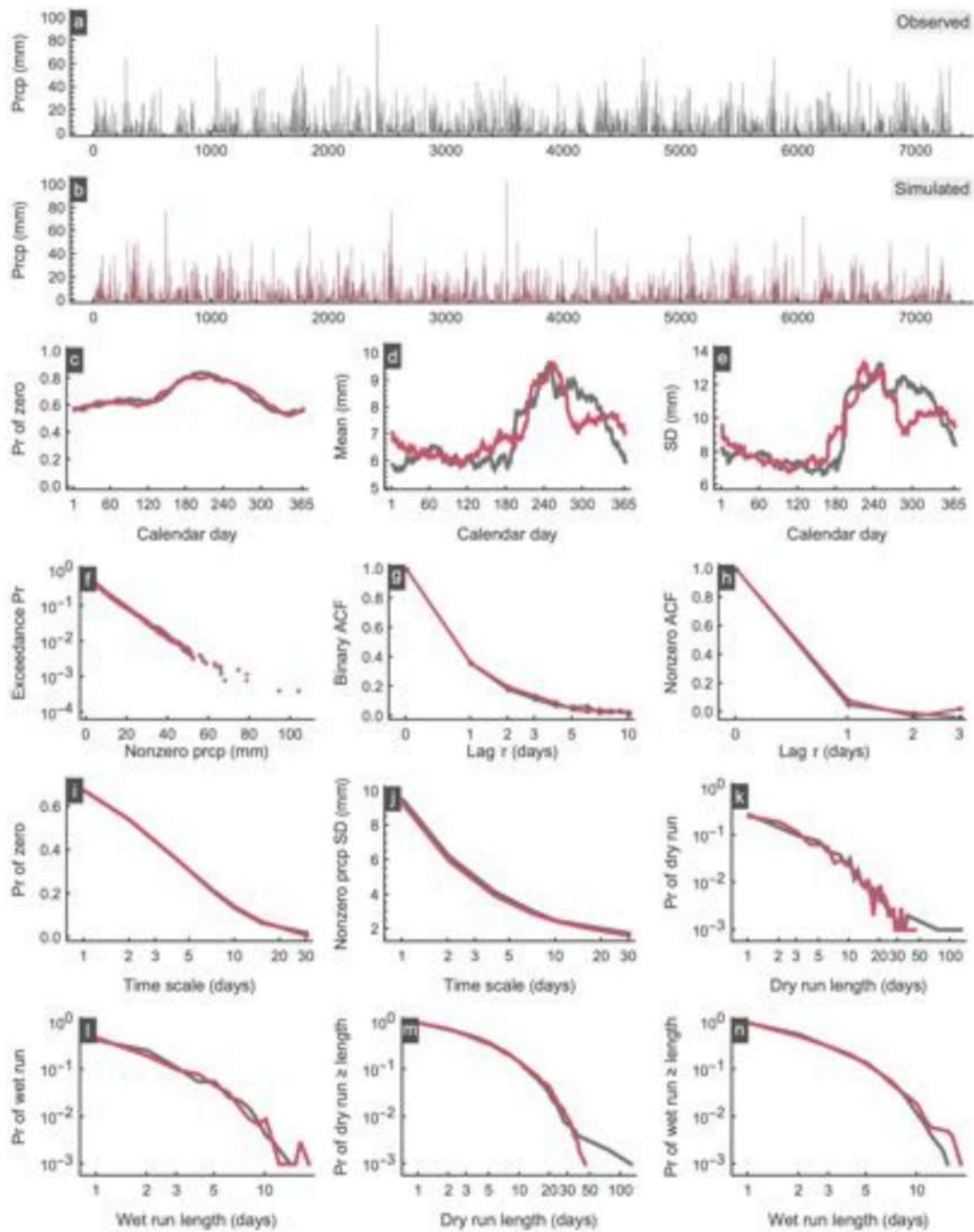


Figure A55. Simulation results for station PluvioMetro 63.

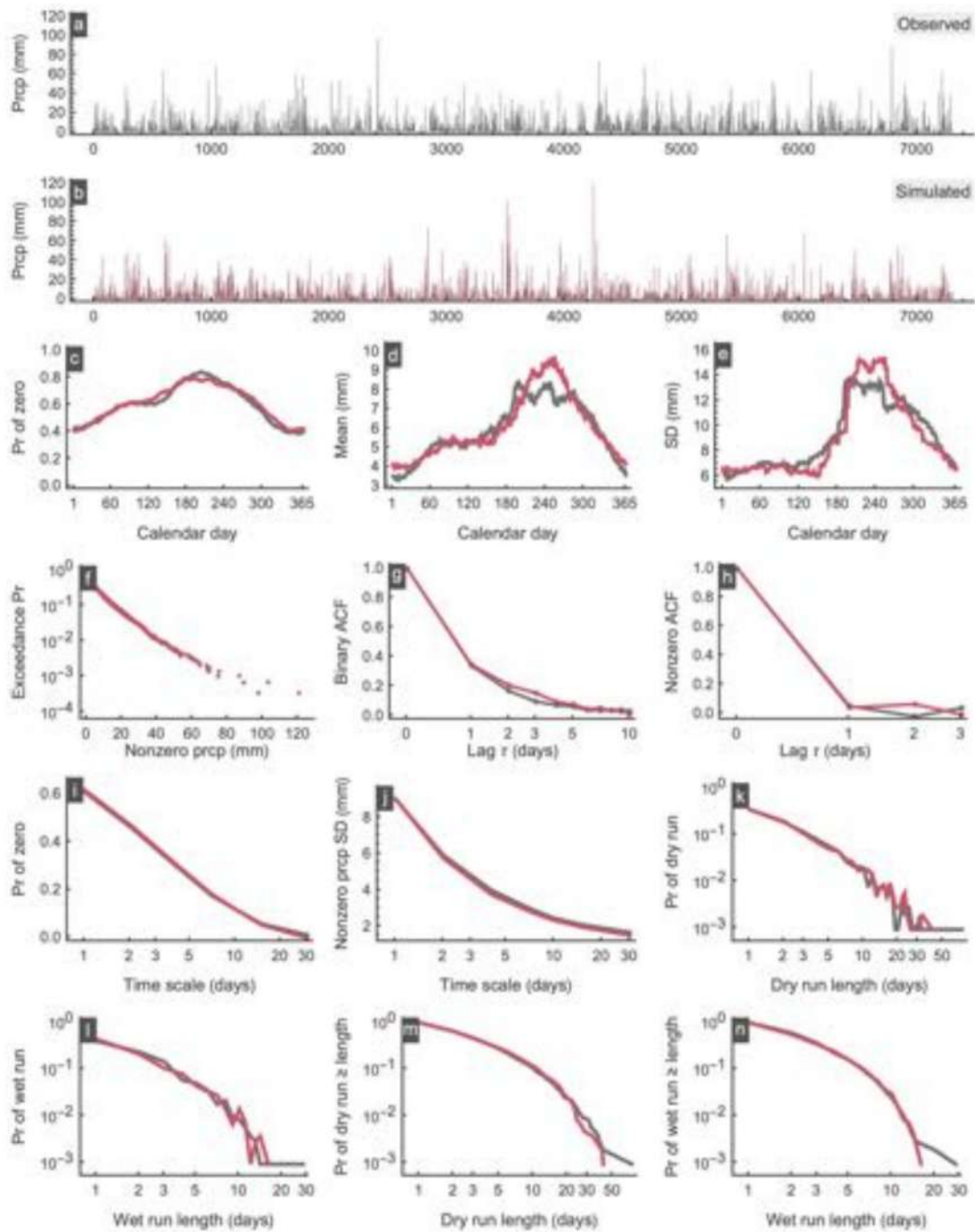


Figure A56. Simulation results for station PluvioMetro 64.

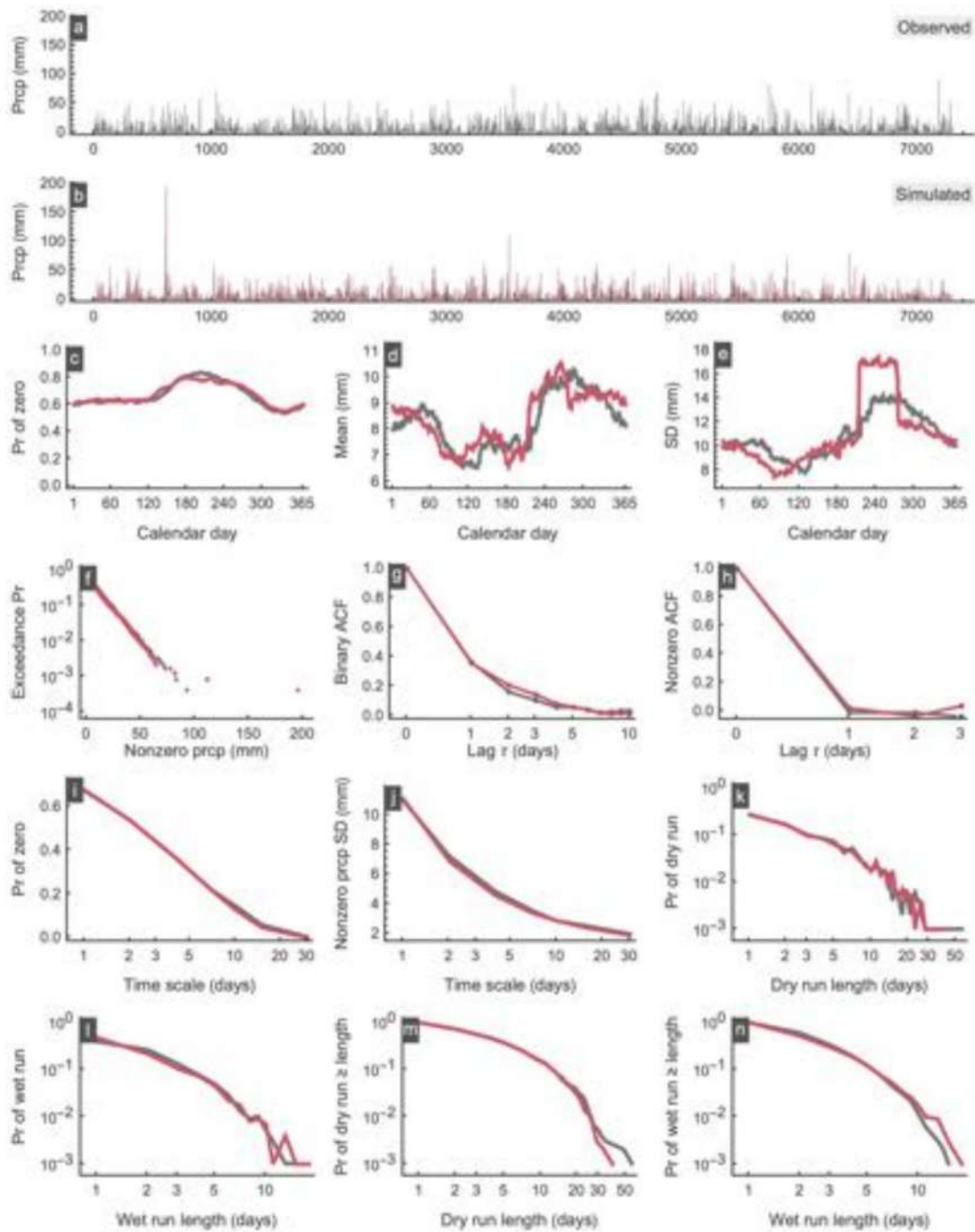


Figure A57. Simulation results for station PluvioMetro 65.

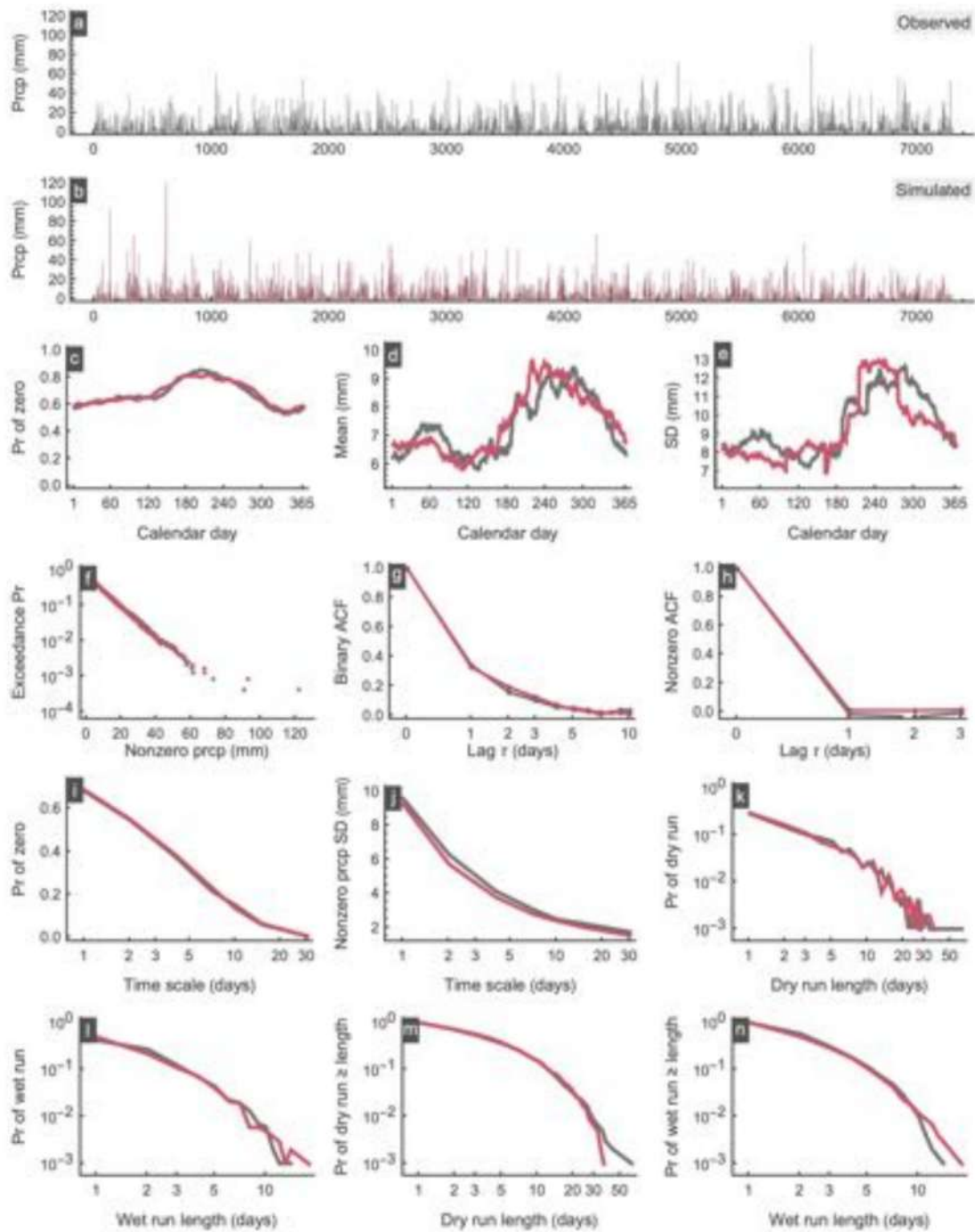


Figure A58. Simulation results for station PluvioMetro 66.

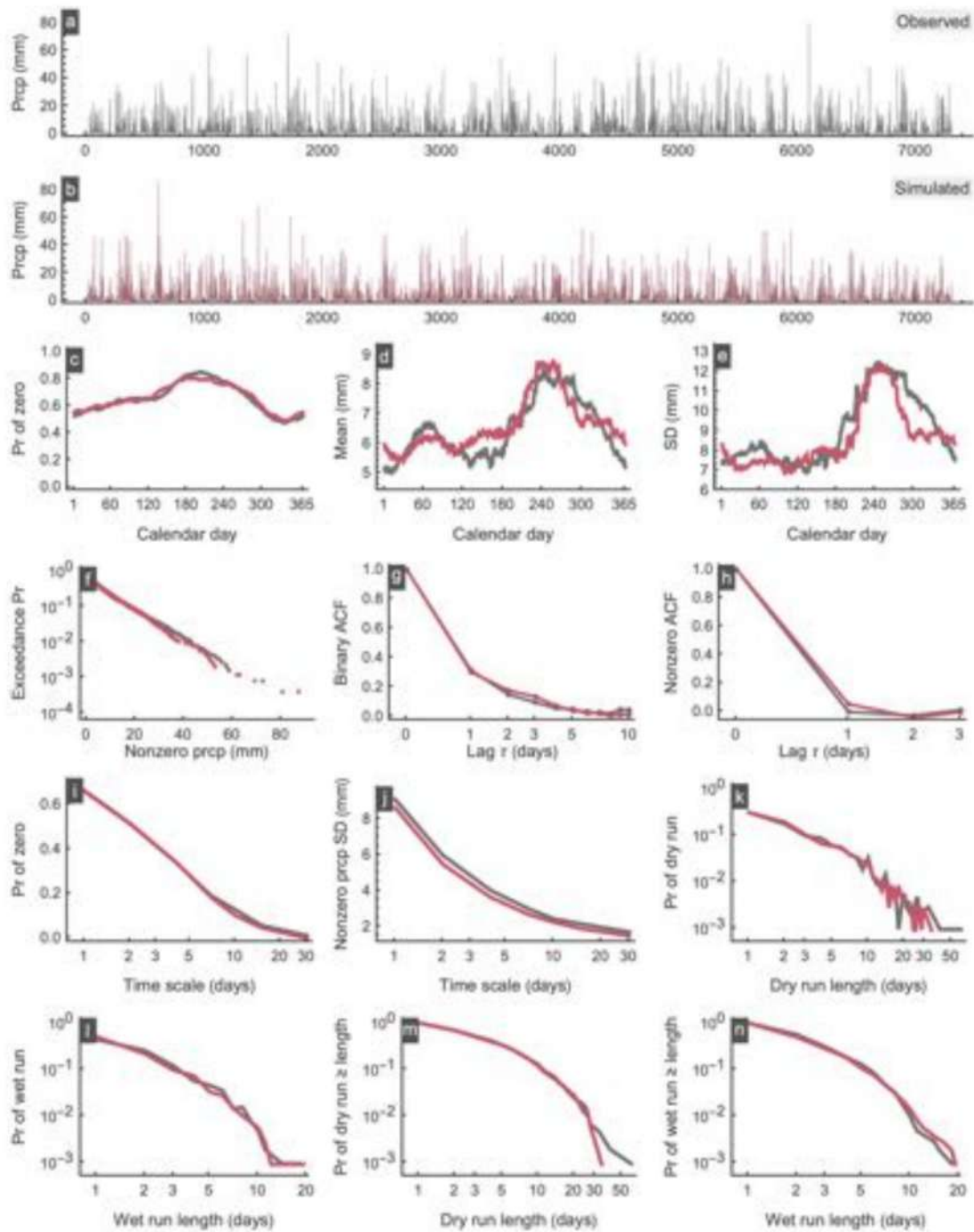


Figure A59. Simulation results for station PluvioMetro 67.

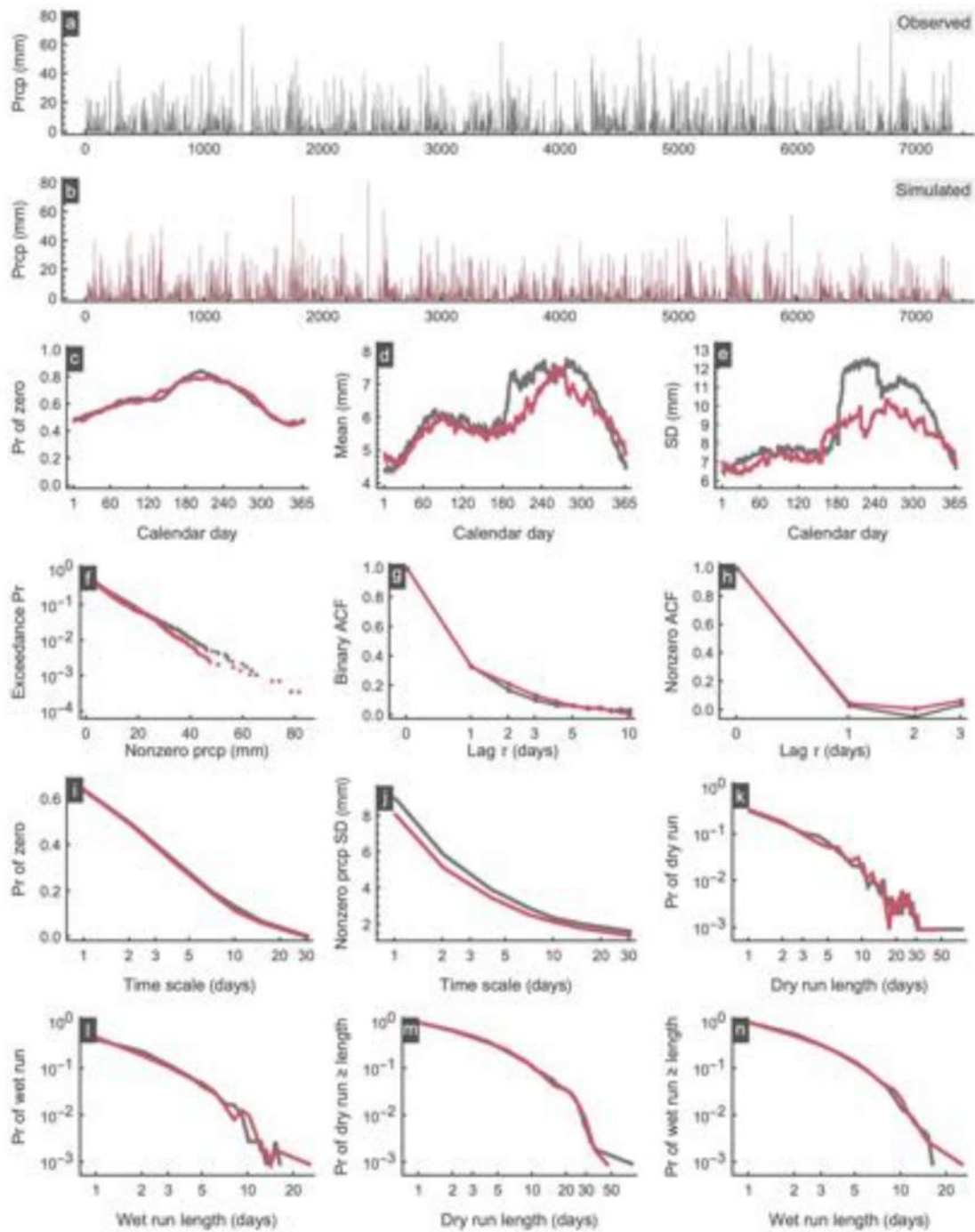


Figure A60. Simulation results for station PluvioMetro 68.

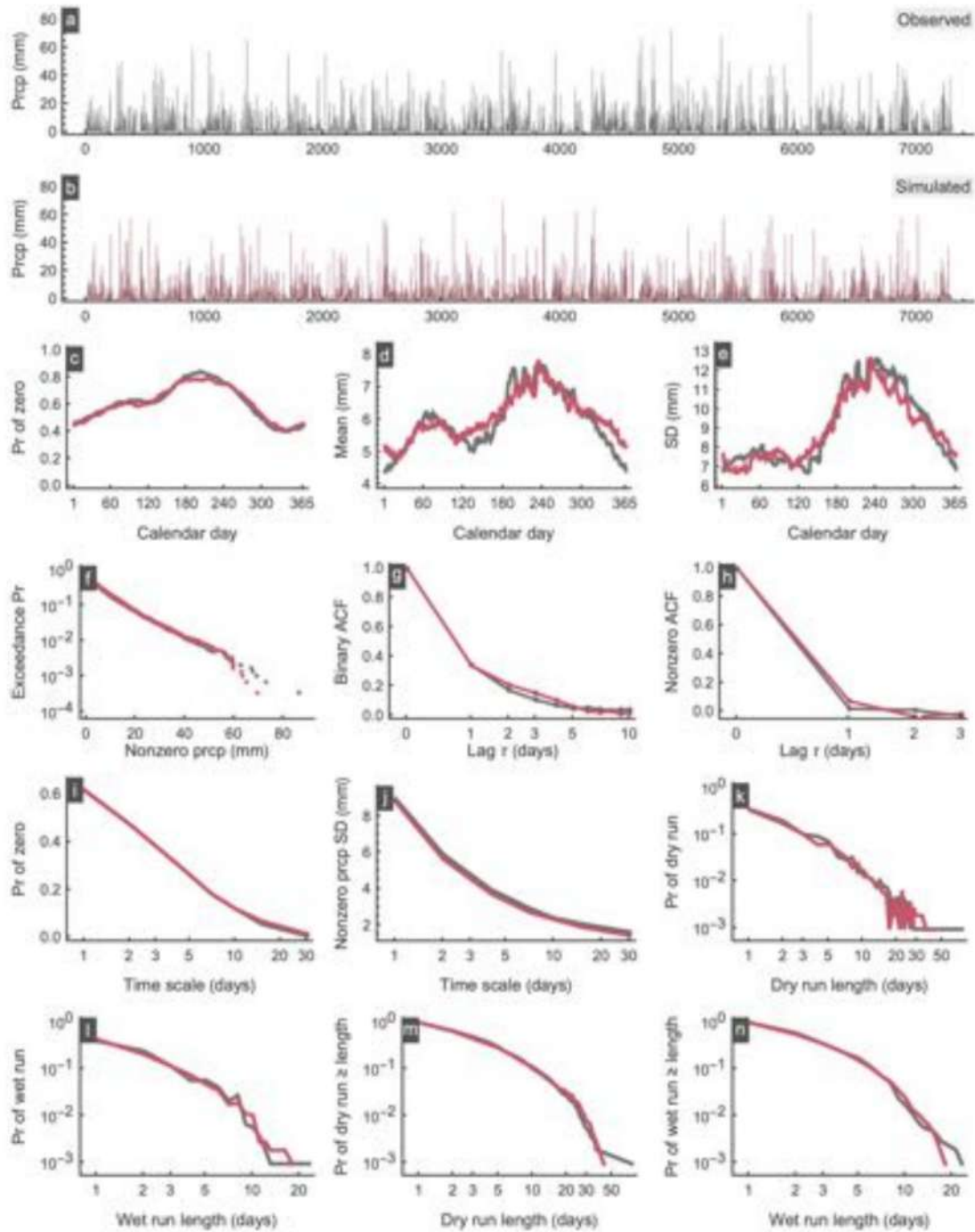


Figure A61. Simulation results for station PluvioMetro 69.

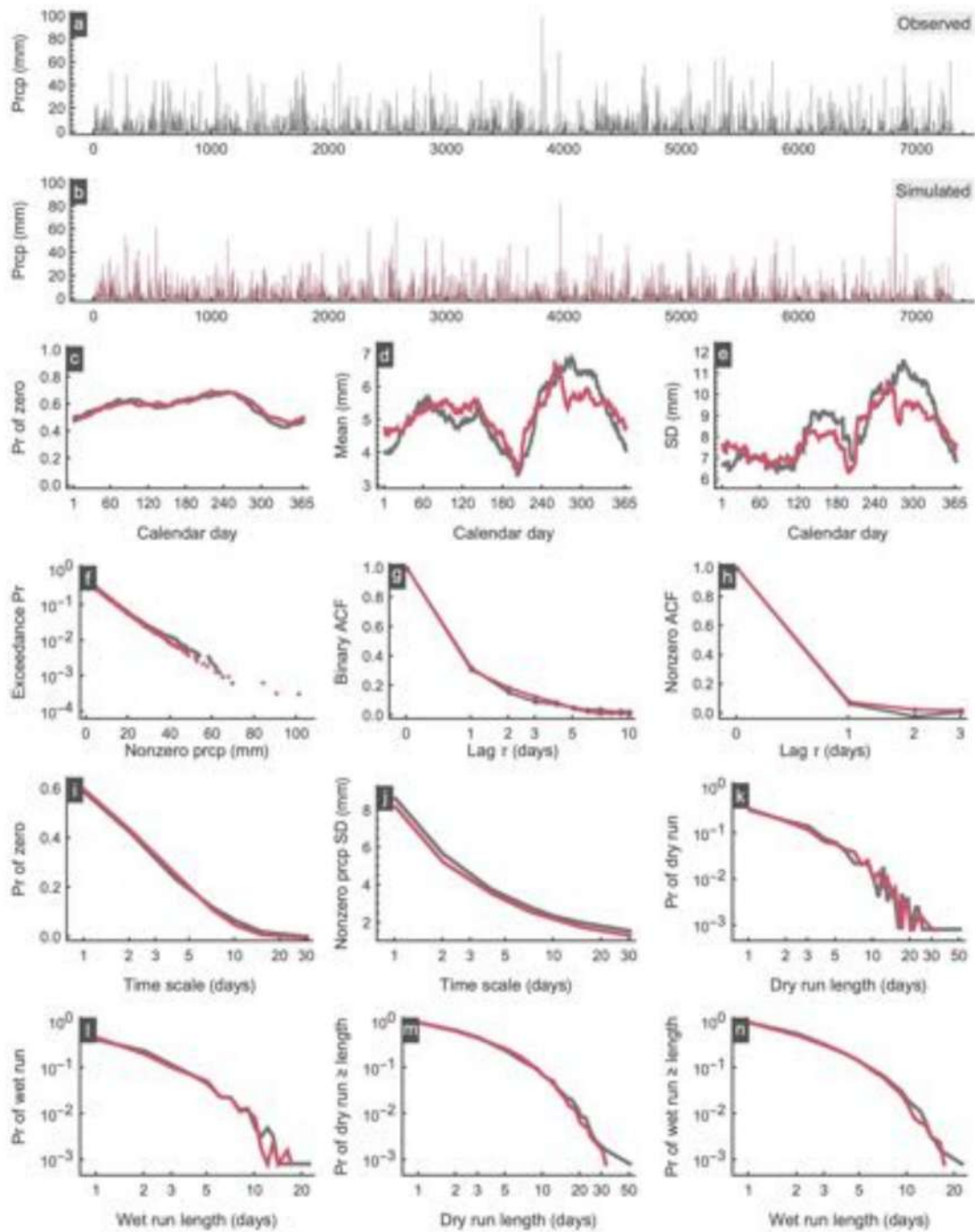


Figure A62. Simulation results for station PluvioMetro 70.

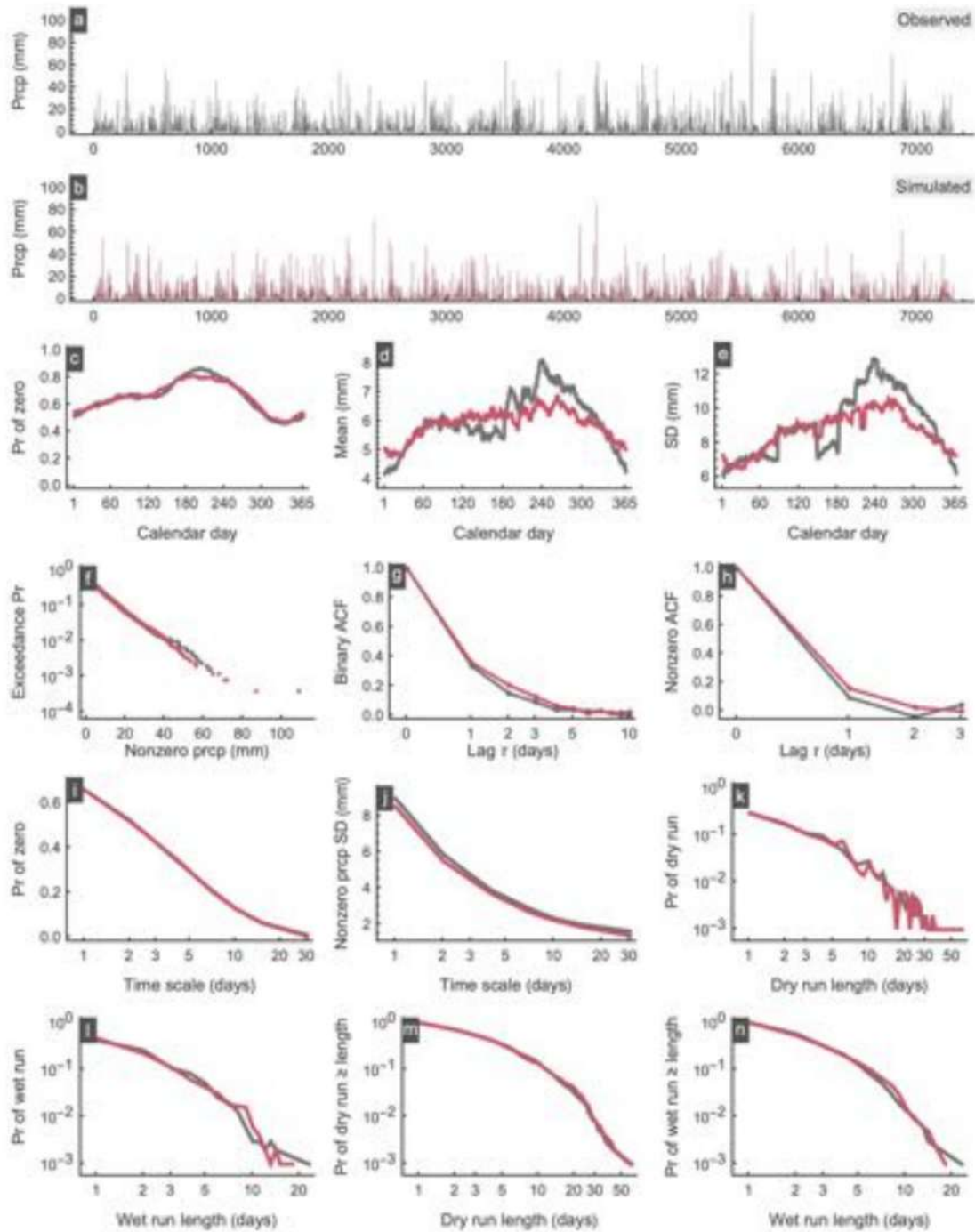


Figure A63. Simulation results for station PluvioMetro 71.

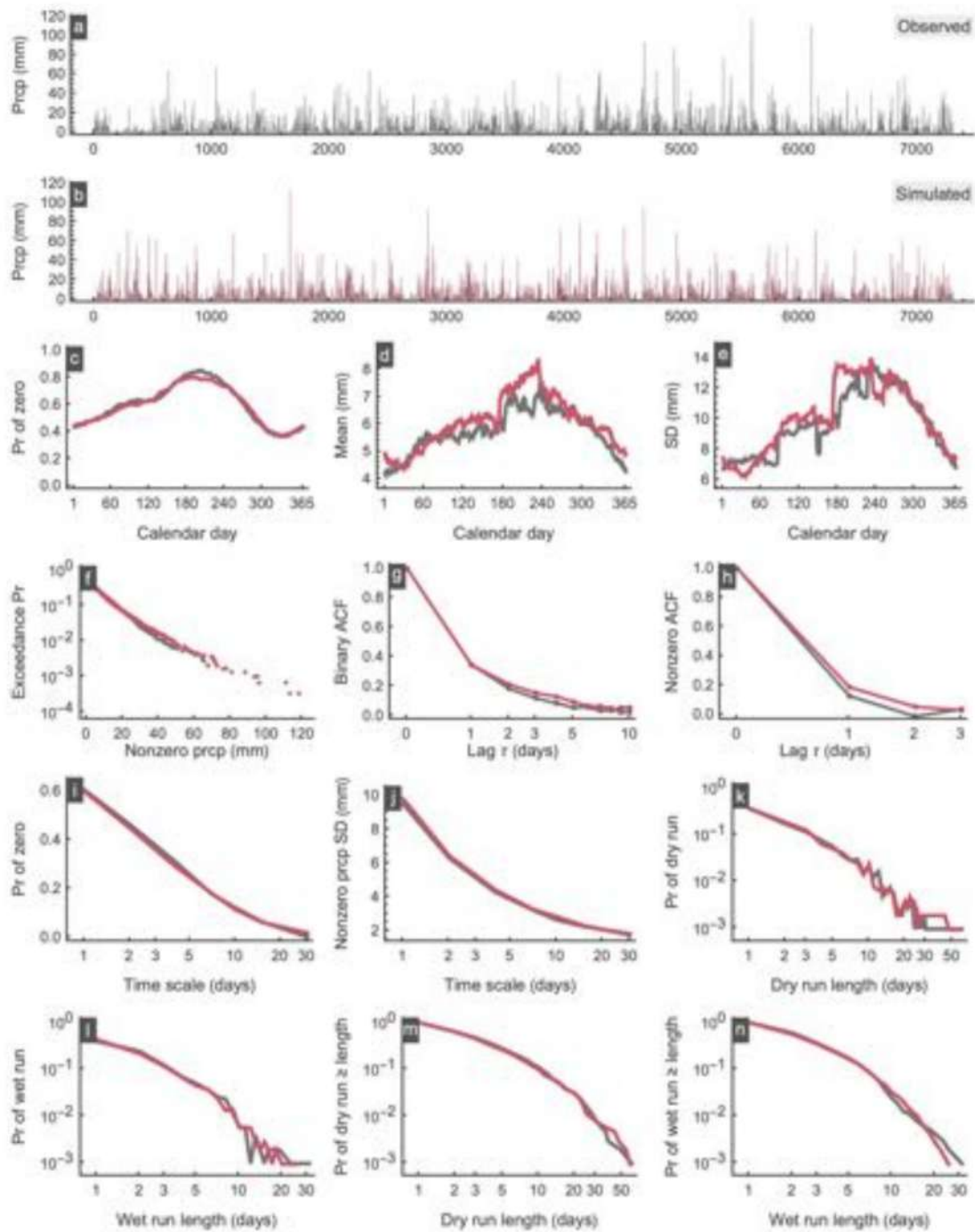


Figure A64. Simulation results for station PluvioMetro 72.

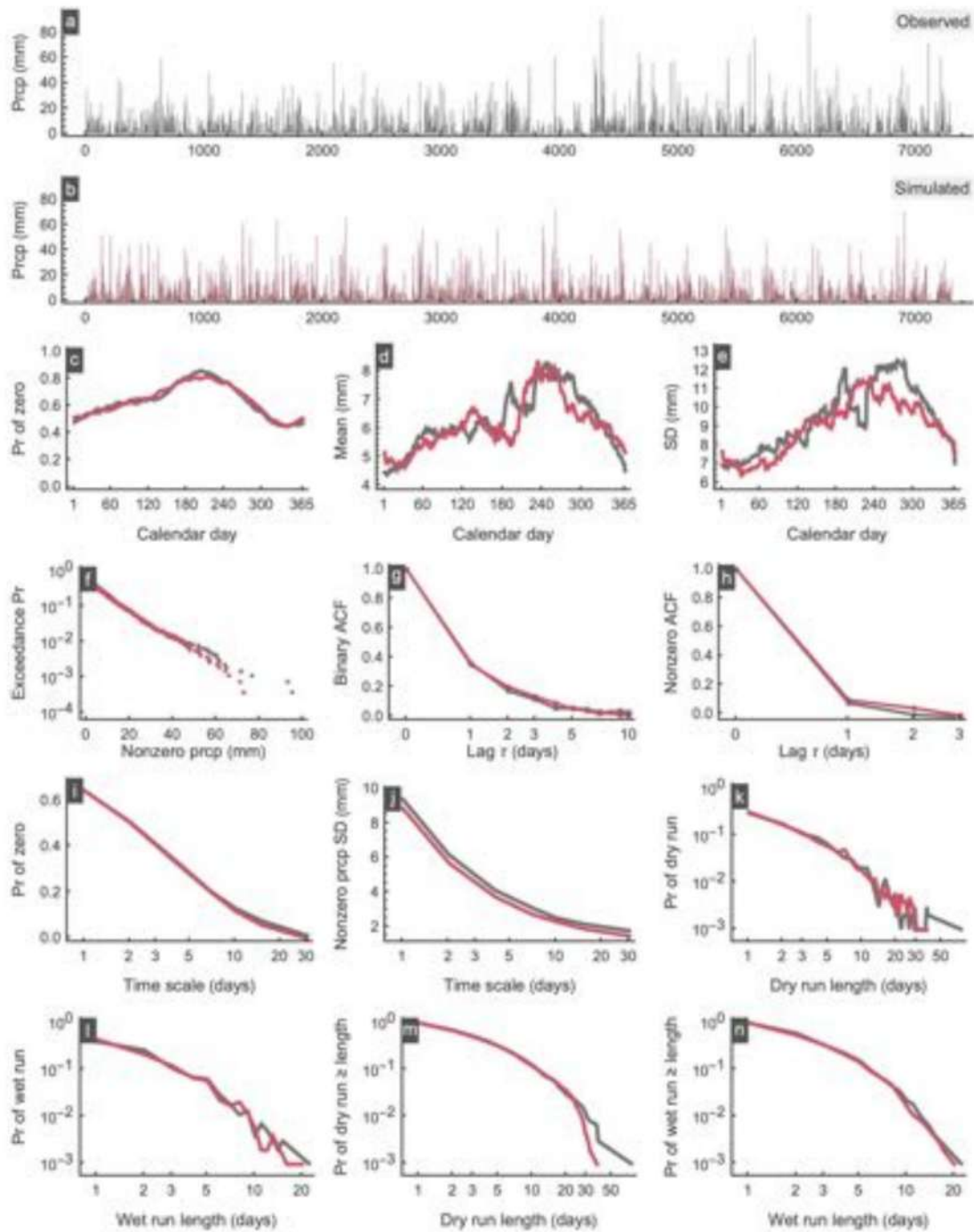


Figure A65. Simulation results for station PluvioMetro 73.

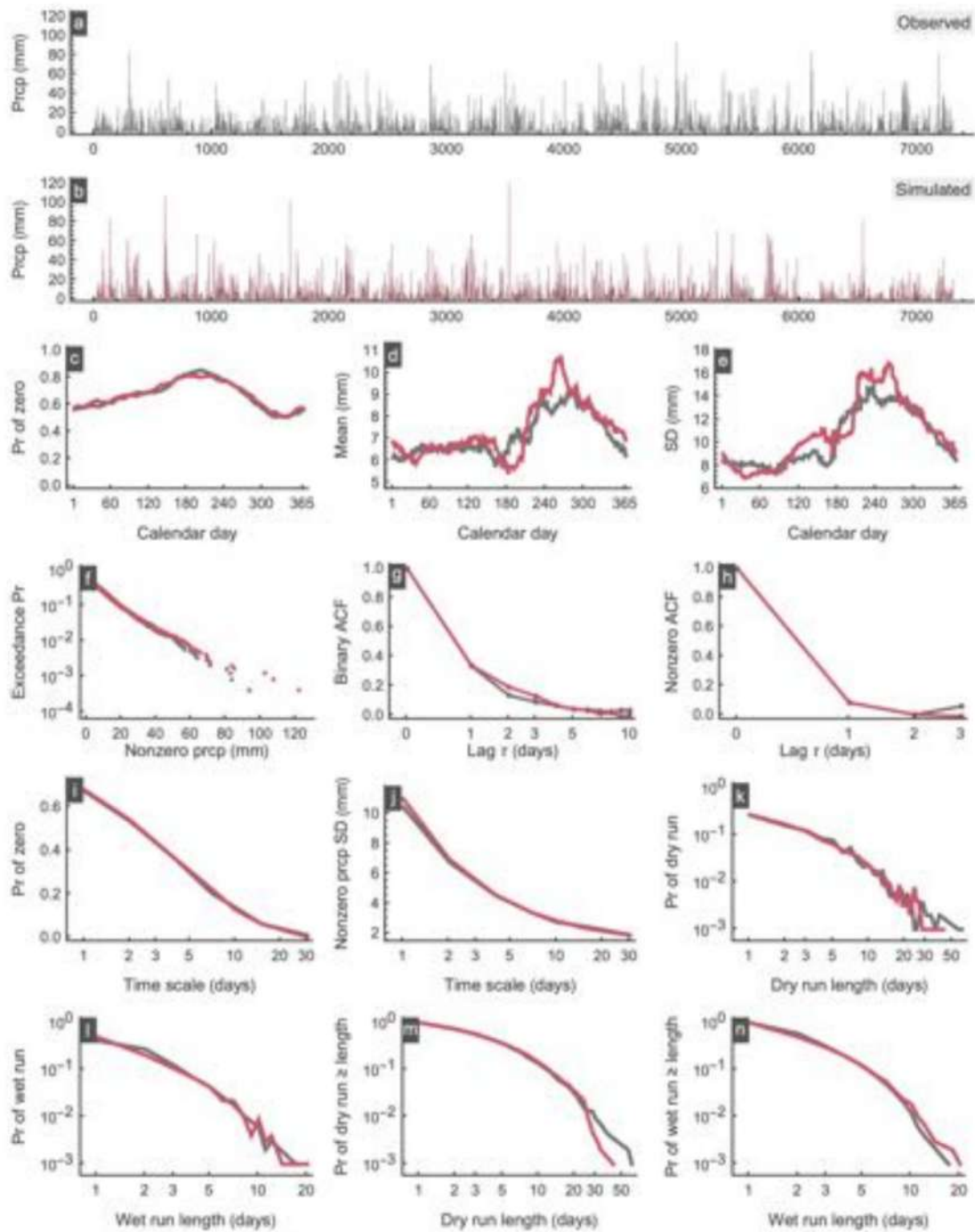


Figure A66. Simulation results for station PluvioMetro 75.

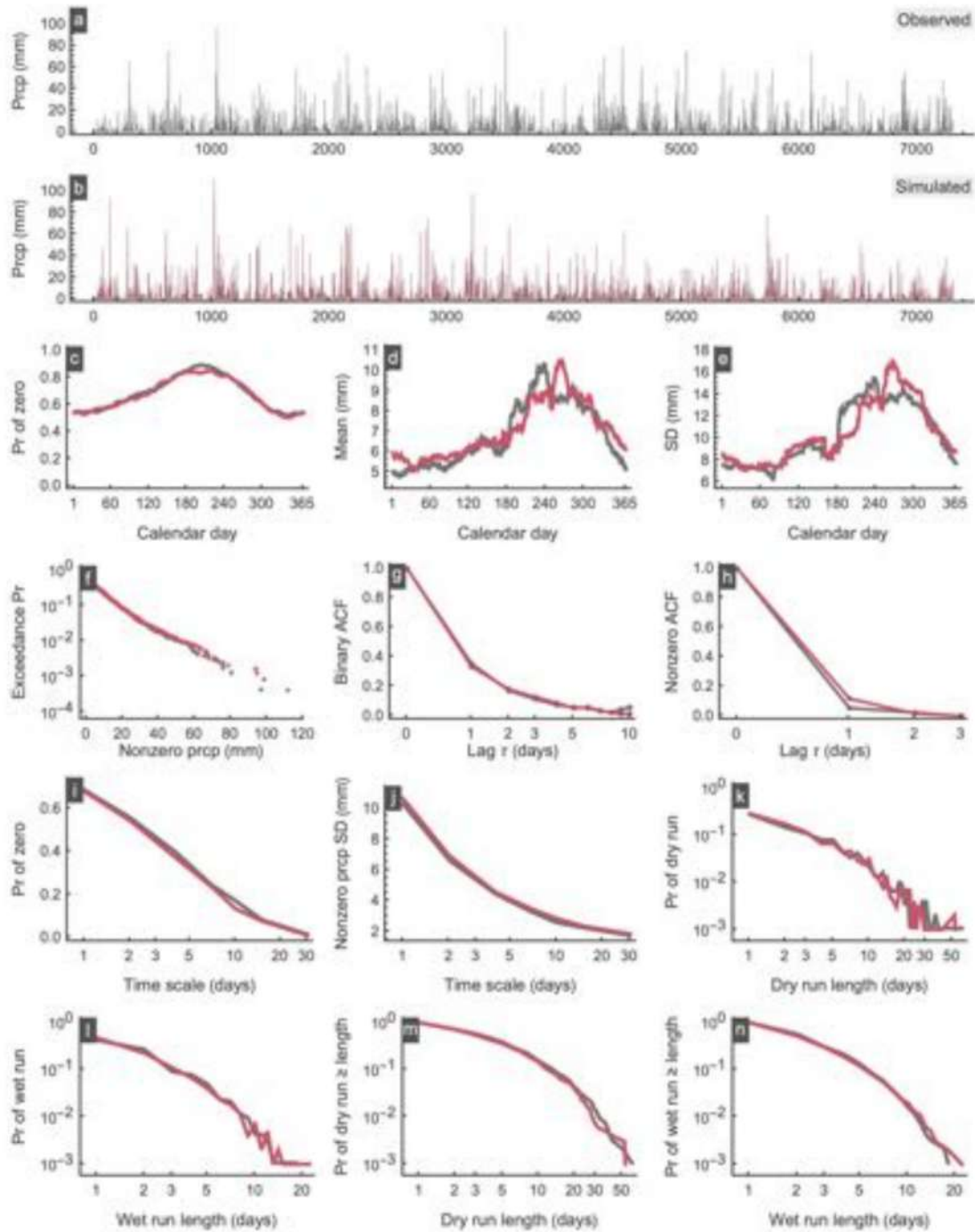


Figure A67. Simulation results for station PluvioMetro 76.

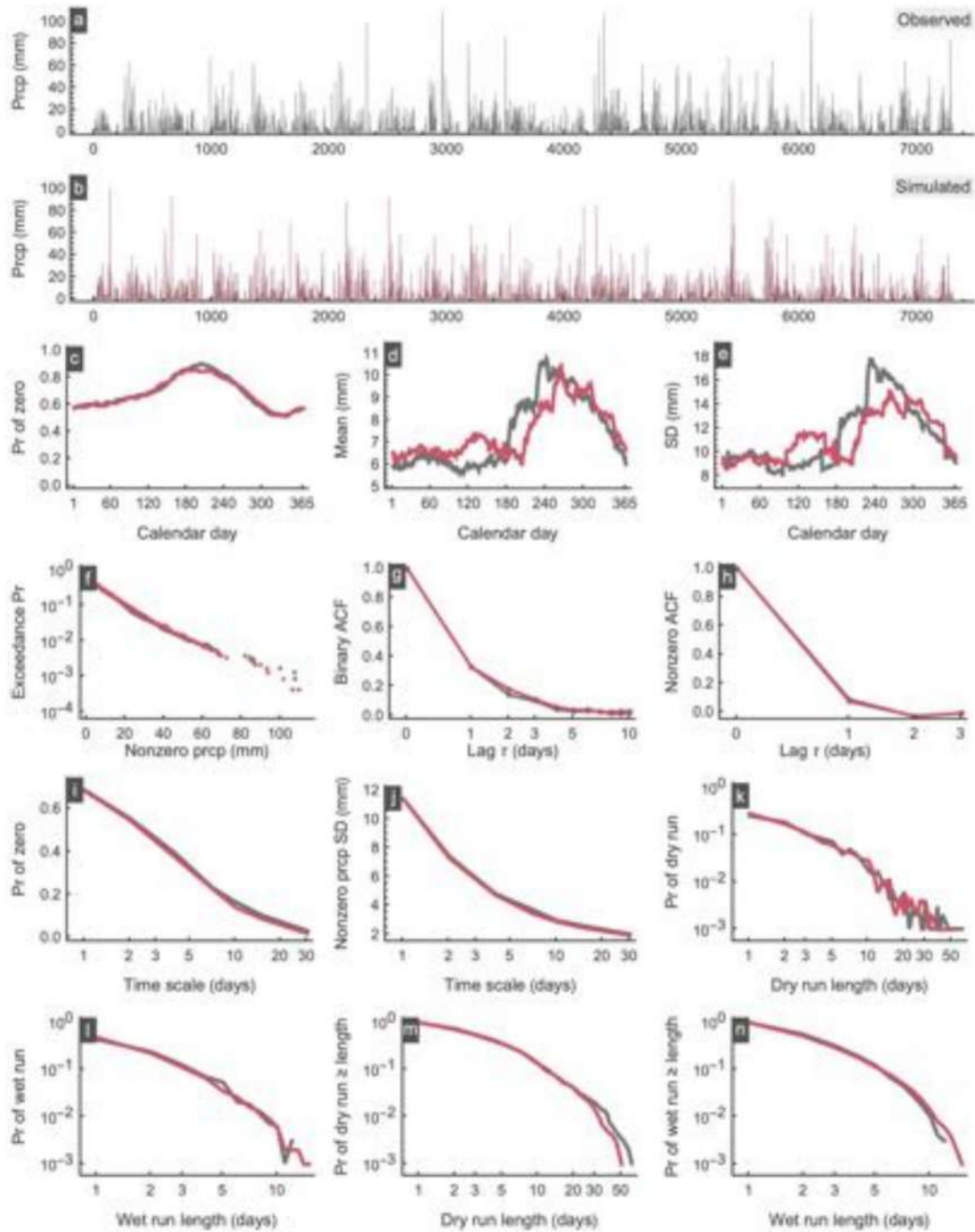


Figure A68. Simulation results for station PluvioMetro 78.

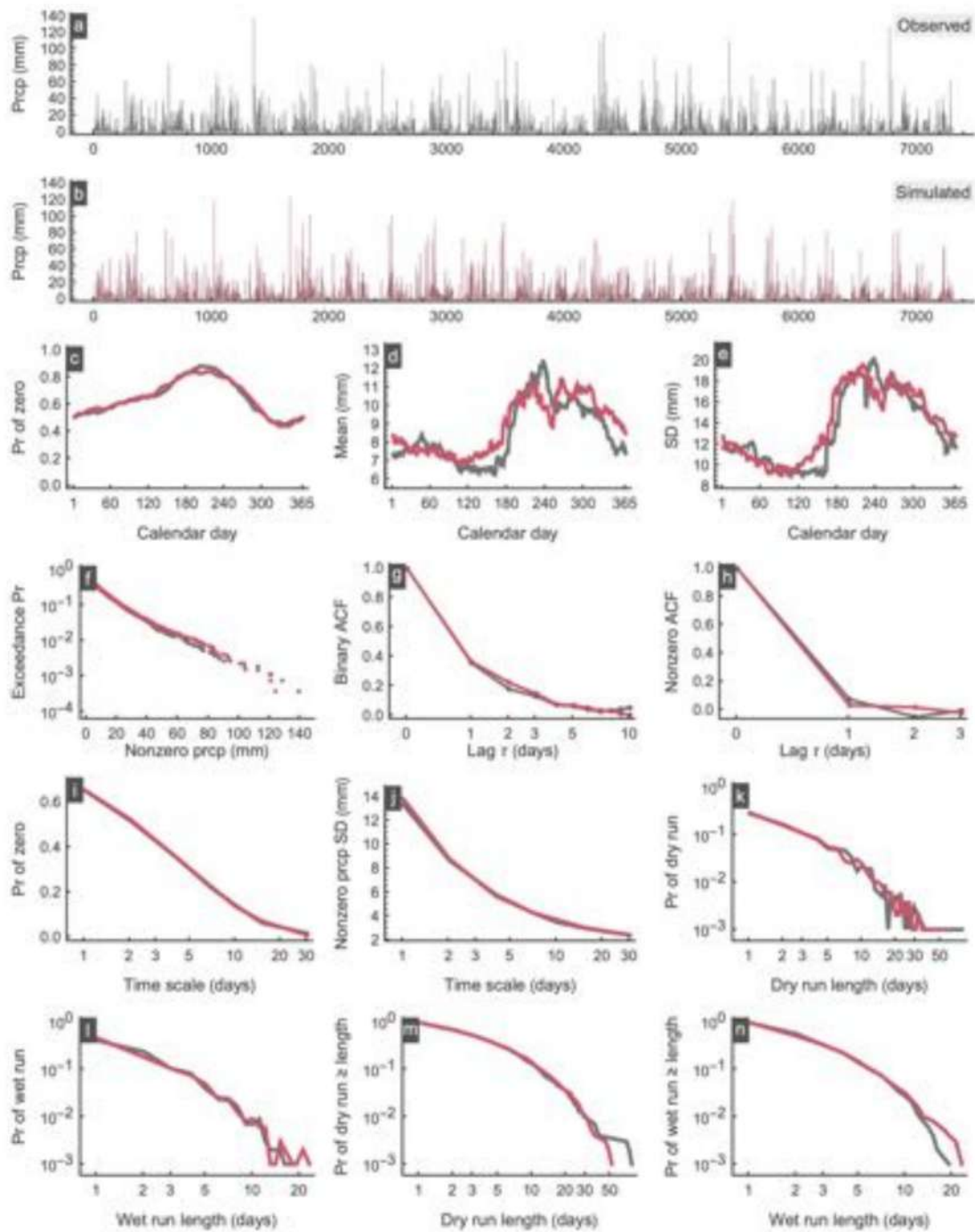


Figure A69. Simulation results for station PluvioMetro 79.

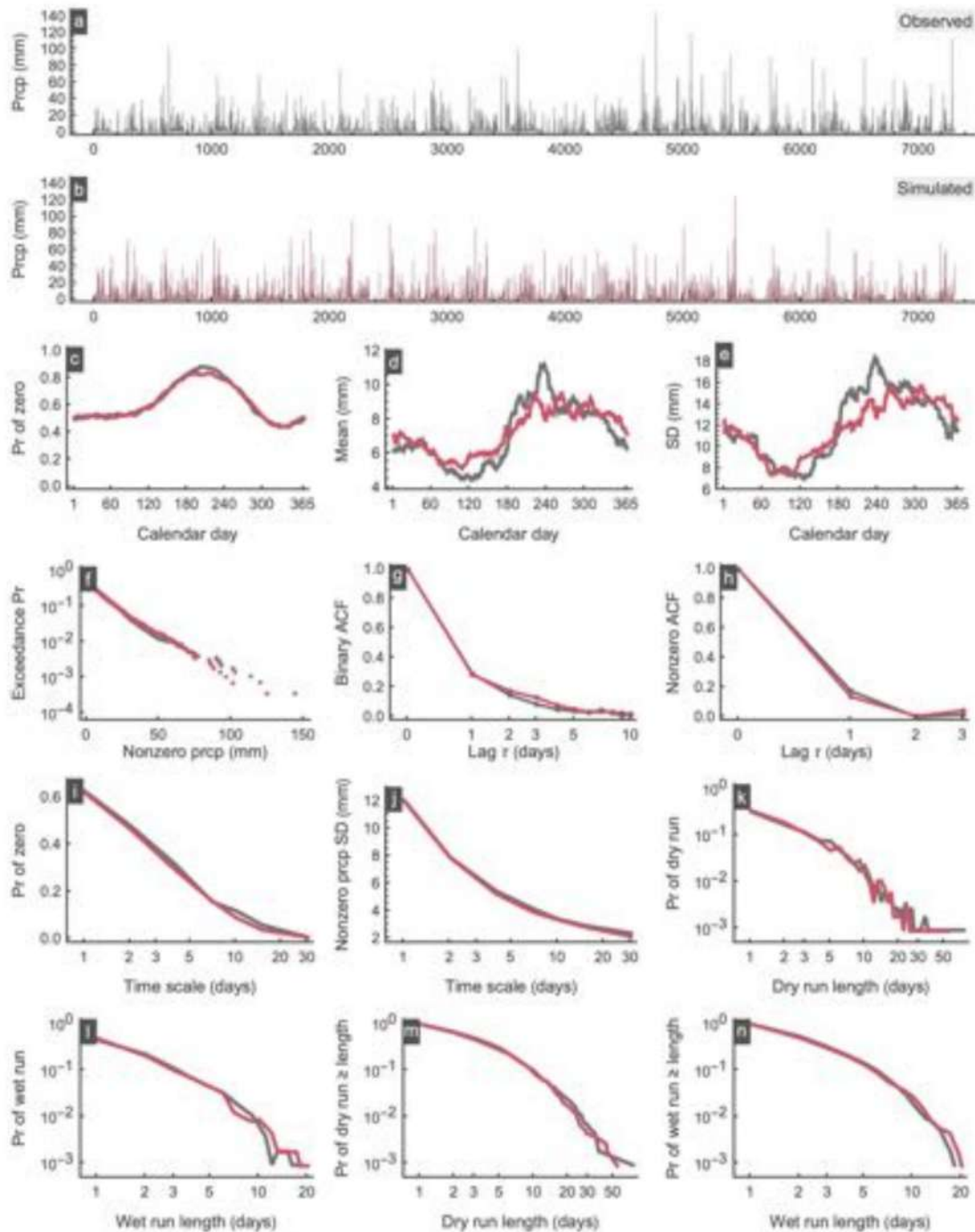


Figure A70. Simulation results for station PluvioMetro 80.

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